MULTI - PHASE (n - PHASE) SYSTEMS : DYNAMICAL MODELLING AND ANALYSIS

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DEDICATED

TO

LORD SHIVA

CERTIFICATE

It is certified that this work, 'MULTI-PHASE (n-PHASE) SYSTEMS : DYNAMICAL MODELLING AND ANALYSIS' by Shri Madan Mohan Choudhary has been carried out under my supervision and it has not been submitted elsewhere for a degree.

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M.M. Choudhary

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A_m	System Matrix
v_d, v_q	d- axis and q- axis stator voltages
v_F	field winding voltage
E_{FD}	field voltage referred to stator
M	Inertia constant of machine in Sec/radion/sec.
K_D	Damping coefficient of Machine
δ	Instantaneous rotor angle
ω	Instantaneous rotor angular speed in rad/sec.
n	no. of phases
P	operator d/dt
Z_P^n	phase impedance matrix of n-phase element
V_P^n, I_P^n	voltage and current vectors of order n
α_{3t}, α_{nt}	variable turn rations of transformer winding on Three-Phase and n-Phase sides respectively.
η	Transmission Line Efficiency
$Z_{P,eq}^n$	n-phase equivalent impedance matrix
A^n, B^n, C^n, D^n	ABCD Parameters of n-phase System
I	Identity Matrix
$L_{aao}, L_{aa2}, L_{abo}$	Inductive coefficients of the stator winding

T_m, T_e	Mechanical and Electrical Torques of a Synchronous Machine
H	Moment of Inertia in Sec.
L_d', L_q'	Direct and quadrature axis transient inductances of Synchronous Machine
L_d'', L_q''	Direct and quadrature axis subtransient inductances of Synchronous Machine
T_{do}', T_{qo}'	Direct axis and quadrature axis transient open circuit Time constant of synchronous machine
T_{do}'', T_{qo}''	Direct axis and quadrature axis subtransient open circuit time constant of synchronous machine
T_d', T_d''	Direct axis transient and subtransient short circuit time constants of synchronous machine

Subscripts

o	suffix to denote the quantities about an operating point
Δ	Prefix to denote small changes in the quantities about the operating point
t	transpose of a matrix or a vector.

SYNOPSIS

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MULTI-PHASE (n-phase) SYSTEMS : DYNAMICAL MODELLING AND ANALYSIS

The local area demand of electric energy was previously met by a locally constructed generating station known as urban system. However, gradually the power stations with higher capacity were installed which have been supplying the electric energy covering a wider area. Steadily with the installation of bigger plants, such as superhydel and superthermal plants near the source of energy for economic considerations, the electric energy is now being transmitted to the load centres far off from the generating stations. Thus the problem of electric utilities has been to transmit large amount of power economically and efficiently. Although the solution to this problem has been achieved to some extent by increasing the transmission voltage which ensures greater economy, reduced losses and improved regulation, yet the three-phase transmission system at the increased transmission voltage, for example, at EHV/UHV levels, presents certain drawbacks, such as diffi-

culty in acquiring new rights-of-way, high surface voltage gradient on conductors, strong electric field on the ground surface with possible biological effects, increased audible noise and radio interference, switching surge overvoltages which cause problem to air gap insulation, problem in insulation co-ordination, increased short-circuit currents and possibility of ferro-resonance conditions. Because of these factors, increasing the transmission voltage seems to have also reached a point of saturation.

Further, HVDC transmission emerged as an alternative which has some advantages, such as increased stability limit, lower losses and costs than the equivalent a.c. lines; it has the advantage of its use as an asynchronous connection which does not raise the fault level appreciably, and it can also be used as a link between a.c. systems having different frequencies. But, it has some disadvantages too, namely higher terminal costs, introduction of harmonics resulting in greater losses and communication interference, and also difficulties in providing adequate protection scheme.

The other alternative may be multi-phase i.e. high-phase order transmission (HPOT) system. The concept of the multi-phase system was initiated during the year 1972 and various studies have been conducted and reported since then.

The multi-phase transmission system has the advantage of acquiring new rights-of-way efficiently and effectively. The general feasibility of multi-phase lines has been investigated and the six-phase line appears to be more suitable than the other multi-phase lines. The feasibility studies of six-phase line obtained by converting double-circuit three-phase lines are also reported to have been carried out. The installation and the field testing of some experimental six-phase and twelve-phase lines are reported to have been undertaken. The overvoltages and insulation requirements for six-phase lines have also been studied.

The various elements of six-phase systems for the purpose of steady-state analysis have been mathematically represented. The model of six-phase synchronous machines in phaser co-ordinates is reported for the analysis under balanced as well as under unbalanced condition.

The mathematical model of three-phase/six-phase transformers with different primary and secondary connections with and without leakage impedances are available in the literature, and based upon these models, the three-phase and the single-phase equivalents of a six-phase transmission line integrated with two three-phase/six-phase transformers have been developed.

The load flow study is reported to have been carried out on mixed 3-phase and six-phase, and completely six-phase systems under balanced as well as under unbalanced conditions. The impact of converting double-circuit 3-phase lines to single-circuit six-phase lines at the same line to line (i.e. adjacent phase) voltage on the load flow performance has been investigated. It has been concluded from these studies that, the mixed three-phase and six-phase system, and a completely six-phase system have the advantage of better voltage magnitude, improved phase angle and increased efficiency, even for the higher system loadings than that of the conventional three-phase systems.

The short circuit analysis, an important study for the purpose of designing protection scheme, is reported to have been carried out in order to investigate the effect of six-phase line in comparision to double-circuit three-phase lines with equal line to line (i.e. adjacent phase) voltage, by deriving appropriate six-phase transformations, such as symmetrical component transformation and Clarke's component transformation.

It is not only the multi-phase transmission (high phase order transmission i.e. HPOT) system, but also the six-phase (Multi-phase) synchronous machines have been examined

for higher rating, employing different types of stator winding arrangements since a long time. The steady-state analysis of a six-phase machine is reported to have been studied with the help of an orthogonal transformation, conceptualizing it as a combination of two sets of Park's transformation.

Although a considerable amount of work on the multiphase transmission and generation system has been reported in the literature, yet the other aspects of investigations, such as transient and dynamic stability studies including modelling for the purpose of such studies have not been undertaken in detail so far. These aspects have been dealt in detail in the present work.

The salient features of the work reported in the thesis are as follows:

1. to develop a detailed dynamical modelling of a six-phase synchronous machine;
2. to develop mathematical representations for various multiphase (n-phase) elements, such as n-phase synchronous machine, 3-phase/n-phase transformers, n-phase transmission lines, of power system networks for steady-state condition;
3. to investigate the dynamic stability of a six-phase synchronous machine and its comparision with that of three-phase machine; and

4. to investigate in detail the load flow, transient stability and dynamic stability with a view to examine the performances of mixed-phase (3-phase and 6-phase) systems and completely six-phase systems, with that of conventional three-phase system.

A brief account of the work reported in the thesis is presented in the following paragraphs.

The dynamical equations of a six-phase synchronous machine have been developed with the help of an orthogonal transformation starting with the inductance calculations of the machine right from the fundamentals. Further, a linearized model in terms of current variables has been developed, and employing the eigenvalue technique, the dynamic stability of a six-phase machine has been investigated and compared to that of a three-phase synchronous machine.

Generalized mathematical representations of the various components of multi-phase (n-phase) power system elements for steady-state condition have been developed. A mathematical model of a multi-phase (n-phase) synchronous machine has also been developed in phasor co-ordinates from which the model of a six-phase machine has been derived. The mathematical models of 3-phase/n-phase transformers for various connections, viz. 3-phase star/n-phase star, 3-phase delta/

n-phase star, have been developed with tappings on the primary as well as on secondary side taking leakage impedance of the transformer into considerations. The three-phase equivalent of a n-phase transmission line has been developed in terms of impedance and ABCD parameters for π representation. This representation is helpful in analysing the system on completely three-phase basis. The equivalent single-phase representation has also been obtained with the help of the three-phase equivalent in terms of ABCD parameters representing π-circuit to analyse a fully balanced system. The n-phase equivalent of a three-phase line connected with a 3-phase/ n-phase transformer at each end of the line, has been obtained. This n-phase equivalent representation will be needed to carry out the analysis of the mixed system completely on n-phase basis (i.e. if the interest of investigation lies on the n-phase part of the system).

The load flow study has been carried out to investigate the impact of converting double circuit three-phase lines to single circuit six-phase or twelve phase lines under two voltage conditions, namely 1) when the phase to ground voltages of the multi-phase line and the double circuit three-phase lines are equal and 2) when the phase to ground voltage of the multi-phase line is $\sqrt{3}$ times that of the double circuit three-phase lines. Two types of cases have been taken into consi-

derations: 1) for a given slack bus voltage; and 2) for a given reactive generation at the slack bus. The single-phase representation of a n-phase transmission line connected via two 3-phase/n-phase transformers has been obtained on the same base quantities as three-phase transmission line.

The direct and quadrature axis transient and sub-transient inductances of a six-phase (multi-phase) synchronous machine have been calculated from the fundamental. The various time constants of a six-phase synchronous machine have also been obtained. The transient stability investigation has been carried out on a sample network for the various system configurations and voltage conditions. In this analysis, the synchronous machine is represented as a constant voltage source behind a direct-axis transient reactance, and the loads as constant impedances. The mixed 3-phase and six-phase system, and a completely six-phase system have been investigated and compa red with conventional three-phase system from the transient stability point of view.

The dynamic stability of a sample network for its different system configurations and voltage conditions has been investigated employing eigenvalue technique based on a linearized model. The two-axis model and the cl assical model of the synchronous machine have been used. The performance of

a mixed 3-phase and six-phase system with the different system loadings has been compared to that of a completely three-phase system. Also, the dynamic stability of a completely six-phase system has been examined with the same system loadings as the conventional three-phase system and also with the increased system loadings.

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTORY REMARKS

Few people could have really thought about the electricity at the time of its invention in the last century for its presently holding such a position of prime importance as a vital means of serving people, especially towards their prosperity because of its manifold applications in the development. The electricity has now been exploited for its use in diversity of applications including domestic, agricultural, commercial and industrial fields ranging from smaller to bigger ones which essentially enhance the industrial and commercial growth with which the individual economy cannot remain unaffected. Although it was initially available for the use of limited localities, and that too in the specific fields, it is now meeting the growing demands with increasing generation. Moreover, electricity is now gradually becoming a kind of necessity. Then, it is but natural to arise a sort of inquisitiveness for its coming to such a stage, and therefore it is necessary to have a look on the growth of electric power, which is mentioned below in brief.

The electricity, an English word, was first used by Sir Thomas Browne in 1646, but William Gilbert Coined in his publication 'De Magnete' in 1600, the word 'electrica' for the

substances which behave like amber (elektron in Greek) which when rubbed with a dry cloth produces static electricity — an idea already enunciated by Thales of Miletus as back as 6th century B.C. Moreover, Gilbert pointed out the difference between the attraction of amber and that of load stone, latter only attracts iron. But the continuous source of electricity was not available till Volta invented Battery in 1800, and it remained the only source till the spate of inventions and developments including the discovery of the relation between electric current and magnetic field in 1831, the electro-magnetic laws of induction and the construction of an electric motor by Faraday in the same year, the first generator and commutator by Pixii in 1832 and 1833 respectively, paved the way for their commercial exploitation by Gramme in 1870.

Near about 1851, there was a great interest in applying electricity as the source of illumination for the light houses and this gave a considerable impetus to the design of magneto-electric machines including that of Saxton, Sturgeon and Stohrer. In 1857 two dc machines with 90 rpm were installed in south fourland lighthouse and the electric light was thrown on the sea for the first time in 1858. In 1873 Gramme installed his first machine for arc lighting to replace the old generator in the french lighthouse, and at the sametime, he build several generators to replace batteries. In 1878 Edison devised a platinum wire filaments, and by 1882 he installed over 150 plants

furnishing power for 3000 incandescent lamps from Edison's Jumbo dc dynamos driven by steam engines.

AC systems were being developed in Europe during 1880s; one of the most successful was that of Lucien Gauland and J.D. Gibbs of Paris who first demonstrated their system in London in 1881, and in the same year Savoy theatres Grossnover Gallery with 2-kV ac machine was equipped. In USA, the first experimental ac distribution system was installed in 1885 which supplied 150 lamps in the town of Great Barrington, Massachusettes The first single-phase ac transmission line was put into operation in 1890 to transmit electric energy generated by water power to a distance of 13 miles from Willamette falls to Portland (oregon), USA, operating at 3.3 kV with a frequency of $33\frac{1}{3}$ c/s.

In 1891 the three-phase long distance ac transmission line from Laufen to the Frankfurt electrotechnical exhibition in Germany at 30-kV for a length of 109 miles was a convincing and successful demonstration. The two-phase ac distribution system was demonstrated to the public at the Columbian Exposition in Chicago in 1893 where the first Large Westinghouse alternator at a frequency of 60 c/s was shown and thereafter, the three-phase ac transmission gradually replaced dc system. In 1894, there were five polyphase generating plants in USA of which one was two-phase and the remaining four three-phase. In Great Britain the earliest three-phase generation and transmission was installed in 1900 at Wood Lane Station transmitting power at 6.6-kV.

During 1906-1911 the transmission voltage in USA mounted from 13-kV to 150-kV. In 1921 the first 220-kV came into operation. In 1936 a transmission line with 287-kV from Hoover Dam to Los Angles was installed. The first 345-kV line in USA was commissioned in 1953; 500-kV line in 1965; 765-kV line in 1969 and the transmission lines upto 1500-kV is considered for future, though feasibility studies have been conducted upto 1100-kV only by BPA (Oregon), USA.

Electric systems were virtually operated till 1917 as individual units because they started as isolated systems and spread out only gradually to cover the wider area. The demand for large blocks of power and increased reliability suggested the interconnection of neighbouring systems.

As the function of a power system is to generate electrical energy economically and without the ecological disturbance, and to transmit this energy over transmission lines and distribution networks with the maximum efficiency and reliability for delivery to consumers located at far places at virtually fixed voltage and frequency, the research regarding this went through various stages to achieve this objective with the investigations examining mainly different aspects concerning the subnetworks, viz. generation, transmission and distribution subnetworks. But this is not very easy because of the rising number of users on one hand, and the technical and the economic limitations on the other. However, the efforts have

gone to increase the generating capacity and simultaneously transmit the power to the point (i.e., place) of utilization. The 3-phase generation and transmission is already established through the research and development and is working satisfactorily. This is because of various advantages offered by a three-phase over a single-phase ac system, such as a single three-phase synchronous generator occupies less floor areas, costs less and also needs much less maintenance in comparision to three single-phase synchronous generators, and three-phase transmission is certainly more economical, efficient and reliable compared to single-phase transmission system. However, with the growing size and complexity of the modern power systems, the main problem for the electric utility has been to increase the power transfer capability of the transmission lines so as to cater to the needs at the load centres far off from the generating stations. The electric utilities have found its solution by increasing the transmission voltage of the three-phase transmission line. But the uprating of the three-phase transmission voltage also seems to have reached a point of saturation because of its inherent drawbacks, such as increased audible noise level, increased voltage gradient on the conductor surface and the strong electric field on the ground with possible bilogical effects, when it is viewed as a means of transmitting more power at EHV and UHV levels and also, it has difficulties in acquiring new rights of way, especially in urban and conjested areas even at HV and EHV levels.

Some of the common concern of electric utilities have been to economise the use of land and to increase the acceptability of overhead transmission lines from the points of visibility and appearance. These problems have been tried to be solved through improved design methods, namely compaction, aesthetic designs, etc., aiming at substantial reduction of sizes of three-phase high-voltage lines. But at the same time, the limitations in acquiring new rights-of-way attracted the attention of power system engineers towards searching its viable alternative that ultimately resulted in the idea of a multi-phase transmission system. This topic of multi-phase i.e. high-phase order (HPO) transmission system has been taken up for the investigations, and the results arising out of several case studies have revealed its considerable advantages over the conventional three-phase transmission system, when the two systems, namely three-phase and six-phase transmission systems are considered for increasing the power transfer capability. Although the feasibility studies of multi-phase transmission system have been carried out and the research are in progress; impact of the conversion of a double-circuit three-phase line to a single-circuit six-phase line has been studied. However, it requires the in-depth investigations of transient and dynamic stability analyses of the power system consisting of multi-phase transmission lines.

From the generation point of view, the efforts towards the larger rating of a synchronous machine have been made as it is justified from the security and economic considerations, for it reduces the operational cost per unit of output. To generate more power from a single unit, a multiphase stator winding has been envisaged, as it occupies less floor area and needs less maintenance compared to a traditional three-phase synchronous machine.

The advantages of a six-phase transmission system over the conventional three-phase transmission system reported so far, coupled with the idea of six-phase transmission lines having the same number of conductors as those of double-circuit three-phase lines, and an inquisitiveness towards the querry whether a six-phase transmission line will really serve the purpose of increasing the transmission capability of overhead lines in a better way than a conventional three-phase line along with an intention to study the performance of a six-phase synchronous machine with respect to that of a conventional three-phase machine have indeed provided us with a motive to find out the answers to the querries, and as a result, this topic of multiphase system has been taken for its transient and dynamic stability investigations, which are **relatively** untouched, along with appropriate mathematical modelling.

The power transfer capability of a transmission line, if it is viewed with respect to a double-circuit three-phase

line, can be increased either by increasing the transmission voltage of a three-phase line or by converting the double-circuit three-phase line to a single-circuit six-phase line with equal line to line (i.e., adjacent phase) voltage which will be better out of the two options can be adjudged if the analysis is carried out on both the systems. The steady-state analysis has revealed several advantages of the latter option (i.e. single-circuit six-phase line) with respect to the former. It is important to observe their performances from the stability point of view as well.

Indeed, the stability is a single entity which expresses the ability of the machines to keep the relative angular positions of the rotor constant, if any sort of disturbances is experienced by the system. In power systems, two types of stability are encountered, namely the dynamic stability and the transient stability, depending upon the magnitude and the duration of the disturbance. The power system which is practically a very large and complex networks, always experiences small disturbances; the stability study caused by such disturbances is termed as dynamic or operating point stability study. The other, transient stability study, is carried out when the system experiences large disturbances, such as faults on a high voltage transmission line, etc., for a short duration.

For the investigation of any system, it is essential to develop the suitable mathematical model as a realistic analytical tool to evaluate the system performance; so is true with the multi-phase systems, and the present work is an attempt in this direction by developing suitable mathematical models to carry out the dynamic and transient stability investigation of multi-phase systems in order to observe its performance in comparision to that of three-phase systems.

Before the work carried out in this thesis is presented, a brief account of the various aspects of investigations on the multi-phase systems reported so far in the literature has been highlighted under the following section.

1.2 LITERATURE SURVEY

An idea of multi-phase (more than three phases) transmission system i.e., high phase order transmission (HPOT) system was first presented by Barnes and Barthold[1] by investigating various aspects, such as voltage gradient on conductor surface which is important from the viewpoints of corona inception, electric field intensity on the ground level from environmental viewpoints which is helpful in designing EHV and UHV lines, space energy flow distribution around a conductor which is essential in studying the efficiency of space utilization for power transmission systems, on the different networks of high-phase order ranging from three to thirty six phases

with conductors symmetrically arranged in a circular configuration in order to carry out a comparative study of the different phase order transmission system. It is found that the voltage gradient on the conductor surface for the six-phase system is less than that for the three-phase system while the field intensity on the ground level of the six-phase line is higher. As indicated, the energy flow distribution around conductors is an important index for the effective utilization of rights-of-way, it has been observed that the high-phase order transmission (HPOT) is better than three-phase transmission system in the effective and efficient utilization of rights-of-way; also, it can be used to transmit more power at the increased transmission voltage with the same rights-of-way. It thus attracted the attention of researchers and utility engineers and consequently, the investigations are being carried out since then in order to study further the high-phase system as an alternative to a conventional three-phase system.

A significant amount of research efforts has been applied in investigating the feasibility of multi-phase transmission systems since then, and the conclusions derived from the various feasibility studies have been reported in the literature [1-12] . A systematic and an elaborate study on multi-phase system was carried out under the joint efforts of Allegheny power system and West Virginia University (APS-WVU), and later on, under a project sponsored by the US Department of Energy.

Venkata et al [2] have outlined the concept of multi-phase transmission systems in terms of six-phase lines and have carried out the study of load flow, stochastic reliability and stability analyses by making a preliminary investigation on a 138-kV system under two configurations, namely 1) a completely three-phase system having one double-circuit three-phase line and 2) a combined three-phase and six-phase system by converting the double-circuit three-phase line to a single-circuit six-phase line with 138-kV (phase to ground). It has been shown from the load flow analysis that the more demands can be met from configuration 2 with improved voltage regulation and higher transmission efficiency without overloading the line. Further, two ways of increasing the power transfer capability of a double-circuit three-phase line have been considered: a) increasing the transmission line voltage of the double-circuit three-phase line by $\sqrt{3}$; b) converting the double-circuit three-phase line to a single-circuit six-phase line, the latter having $\sqrt{3}$ times the phase to ground voltage than the former. Although both of these alternatives are electrically ideal, the six-phase transmission line has been shown to require less transmission corroidor [2].

Bhat et al [3] have derived symmetrical component transformation for a six-phase system as a straight forward extension of the three-phase system, and have carried out the fault analysis to determine the fault currents and voltages on

a six-phase transmission system. The feasibility study on the multi-phase system has been carried out by Guyker et al [4] and various aspects have been taken for investigation, and especially, an idea of EHV six-phase line as a viable alternative to UHV three-phase line has been presented.

Stewart and Wilson in their companion papers [5,6] have systematically examined the multi-phase transmission systems in detail. In their first paper [5] , the feasibility analysis has been carried out under steady-state condition on two transmission voltages, for example, 80-kV and 289-kV (phase to ground) for six-phase and twelve phase transmission lines in order to compare them with 138-kV and 500-kV (phase to phase) double-circuit three-phase lines with a view to multi-phase system voltage nomenclature, the effect of transmission line impedance on the steady-state operation, the line and the generator current unbalances and the electrical environmental aspects, such as radio noise, audible noise and the electric fields at the ground; while in their second paper [6] , an extensive switching surge studies arising out of fault clearing on the six-phase lines have been taken for investigation to analyse the fault overvoltages, overvoltage due to interphase coupling and lightning performance to study several aspects of line design, such as the insulation level, the conductor clearances, etc. From the study, it has been shown that the phase to ground voltage is appropriate for system voltage definition for multi-

phase system; the thermal line loading increases proportional to the number of phases while surge impedance loading (SIL) increases at the lesser rate; the generator current and the line current unbalances due to six-phase transmission line with or without transposition are comparable to, or better than those of double-circuit three-phase line; radio and audible noise level are reduced with the increase in number of phases; the electric field at the conductor surface decreases with increasing phase order while the field at the ground level increases; fault overvoltages for a six-phase line can be slightly higher than those for a comparable three-phase line while for the phase order higher than six, the fault overvoltage magnitudes are similar to those of a three-phase line. The interphase coupling has been shown to cause high overvoltages on the open phase when a single-phase switching is applied to high-phase order (HPO) systems with shunt compensation. Further, the phase to phase switching surges have been shown to assume primary importance in the design of high-phase order systems thereby requiring a surge control especially for compact phase spacing. The rate of rise of recovery voltage (RRRV) during fault clearance is less for high-phase order systems than that for three-phase systems with comparable short-circuit MVA. The lightning performance has been shown to be comparable to the three-phase system.

The terminal insulation level for a multi-phase system has been shown to be somewhat higher. However, the typical insulation levels, even for a six-phase system does not exceed to those presently applied on three-phase systems of the same phase to ground voltage.

The fault analysis of a six-phase transmission system has been carried out by Onogi and Okumoto [7] with the help of the combined use of two-phase and three-phase symmetrical component method, and further a very interesting method — connecting two single-phase transformers or one single-phase three-legged transformer for each phase — has been proposed for suppressing the fault current. The fault current has been shown to be suppressed by impedances connected to the tertiary winding, and if the tertiary impedances are infinitely large, the line to ground voltage of the sound phase has been observed to be twice as high as the prefault line to ground voltage and the fault current (either ground fault current or short circuit current) is as small as the exciting current. Two schemes have also been proposed to suppress the overvoltages.

Willems [8] has developed the mathematical model for a six-phase line by ABCD parameters similar to that derived for a three-phase system by obtaining the mathematical description of the various three-phase/six-phase transformers in the phasor co-ordinates without considering their leakage impdeances. Employing these models, the three-phase equivalent representa-

tion of a six-phase line both in impedance and admittance forms and its equivalent ABCD parameters have been derived to carry out the analysis of a composite three-phase and six-phase systems on a three-phase basis. In order to analyse the system completely on six-phase basis, the equivalent six-phase of a three-phase line has been obtained. Further, the equivalent single-phase of the composite system has been derived to carry out the analysis on single-phase basis.

A six-phase transmission line simulator has been developed by Chinnarao and Venkata [9] to study the multi-phase transmission system. Peeran et al [10] have carried out the fault analysis on a six-phase system with the help of Alpha-Beta-Zero component transformation.

Willems [11] has derived the symmetrical component transformation of a six-phase system by considering it as the two coupled three-phase systems, and this concept can be extended to develop transformation for the multi-phase systems that are multiple of three. By this derived component transformation, the impedance matrix of a six-phase transmission line can be diagonalized.

Grant et al [12] have constructed experimental six- and twelve-phase lines achieving a considerable compaction, and choosing a voltage of 80-kV phase to ground (corresponding to 138-kV phase to phase for a three-phase circuit) the electrical

and mechanical tests have been performed demonstrating practical insulator and hardware designs to ascertain the correctness of analytical procedures developed for a high-phase order.

The use of symmetries inherent in power system networks and its exploitation for the purpose of simplifying their analyses was examined by Singh et al [13], and based upon the symmetry considerations alone and using group theoretic techniques, the symmetrical and Clarke's component transformations for the multi-phase systems have been constructed from the fundamentals [14] . These transformations were further extended for the generalized N-Port network [15].

The fault analysis on a mixed three-phase and six-phase system using symmetrical component transformation has been carried out by Nanda et al [16] . Eleven types of faults on the six-phase line have been considered for investigations after obtaining voltage and current relation as well as sequence network connections corresponding to each type of fault. The six-phase and the double-circuit three-phase transmission lines have been compared under two conditions, namely the most severe all phases to ground fault and the most common single-phase to ground fault. It has been found that the fault current involving all phases in the six-phase line is slightly higher than that for the three-phase line, whereas the fault current in case of a single-line to ground fault of the six-phase line is nearly half to that of the three-phase line.

Kallaur and Stewart [17] have examined the uprating to 230-kV (230-kV phase to phase for three-phase operation and 230/ $\sqrt{3}$ - kV phase to ground for a six-phase operation) by selecting five double-circuit three-phase lines. The study has revealed that, among the five specific line designs, one is technically suitable for 230-kV three-phase operation and the three are for six-phase operation while the remaining one is unsuitable for either case. Further, it has been concluded that the phase conversion required for six-phase results in higher terminal cost. However, the benefits of the reduced losses and higher power handling capability may offset more than the cost penality.

The steady-state characteristics viz., voltage gradient on the conductor surface, ground level electric field intensity, space distribution of energy flow density of a six-phase transmission line has been investigated by Takasaki et al [18]. A single-circuit six-phase line with sixteen phase arrangements has been compared to a double circuit three-phase line with two phase arrangements. The study reveals that the field intensity on the ground level in case of a six-phase line is higher than that of a double-circuit three-phase line, while the electric intensity on the conductor surface is lower. It is very interesting to observe that the phase arrangement which gives the smaller voltage gradient on the conductor surface gives a higher field intensity on the ground level.

Stewart and Grant [19] have carried out the studies on the six-phase and twelve-phase test lines, and have confirmed the analytical predictions of electrical and mechanical behaviours demonstrating a simple design and attractive structures built with standard hardware.

Venkata et al [20] have carried out the fault analysis on a 138-kV six-phase line by discussing the various types of fault and their different 23 types of significant combinations likely to occur on a six-phase line, and have developed the analytical expressions for all the 23 types of fault.

Tiwari et al [21] have developed the mathematical model of a six-phase synchronous machine on the same line as that of the three-phase synchronous machine [22] . The mathematical models for the various connections of three-phase/six-phase transformers taking leakage impedances into considerations have been developed in the phasor co-ordinates. Further, the models of a six-phase line integrated with a three-phase wye/six-phase star connected transformer at the each end of the line have been derived in impedance and admittance forms. Moreover, the three-phase equivalent and its ABCD parameters have been developed and from this, the single-phase equivalent has been systematically derived [21] . The six-phase equivalent of a three-phase transmission line has also been obtained [23] .

Although the fault analysis and the load flow analysis for a three-phase system have been reported [24-29] in the phase frame of reference, the load flow investigation on a six-phase line has been carried out by Tiwari et [30,31] for the balanced system on the single-phase basis [30] and for the unbalanced system in the phasor co-ordinates [31] . In the investigation, taking the phase voltage of the six-phase line equal to $\sqrt{3}$ times that of the three-phase line, the same conclusions have been derived as observed in earlier paper [2] , when a double-circuit three-phase line is converted to a single-circuit six-phase line.

Tiwari et al [31] have developed six-phase equivalent of a three-phase line to facilitate the analysis on the six-phase basis and have carried out the fault analysis by utilizing the representation of three-phase elements in the phasor co-ordinates.

The impact of converting a double-circuit three-phase line to a single circuit six -phase line has been studied while carrying out the fault analysis on the combined three-phase end six-phase systems.

Grant and Stewart [32] have examined an alternative to UHV three-phase transmission in which 462-kV six and twelve phase lines and also 317-kV twelve-phase lines have been compared with 1200-kV UHV three-phase transmission of comparable power transfer capability. It has been found that HPO

lines are smaller and require less rights-of-way than their three-phase counterparts. Stewart et al [33] have demonstrated that, for lines longer than a few miles, saving in the line offset the terminal equipment costs and makes HPO an economic alternative. Terminal equipment costs for HPO systems are usually slightly higher than those for UHV three-phase system.

Wilson et al [34] have compared a 1200-kV, three-phase single-circuit line with a 462-kV six-phase and twelve-phase single-circuit lines and have observed that the insulation design complexities increase with the number of phases. For a circular conductor array, the most critical phase to phase spacing due to switching surges has been shown to be normally between adjacent phases. Switching surges on six-phase line are in the same range, or slightly higher than those on a three-phase line. If the switching resistors are incorporated with the circuit breaker, surges on the six-phase line are less than those on the three-phase line. Switching surge magnitudes on six-phase systems for practical combinations of variables, result in transmission line insulation and clearance requirements comparable to those on three-phase lines.

Further, the transients associated with six-phase capacitor switching have been studied by Ramaswami et al [35] on a 138-kV six-phase system by first deriving the analytical expressions and then by getting the results verified by computer

programming. The impact of restrike in a six-phase system has been found to be less than that in a three-phase system.

Guyker and Shankle [36] have presented the study of the terminal equipment design and the results therefrom for the costs of 138-kV double-circuit three-phase line to a 138-kV six-phase or 230-kV double-circuit three-phase operations. A comparision of the total costs shows that the cost of the 138-kV six-phase uprating option is higher than that of the 230-kV three-phase uprating option due to the transformer cost. However, 138-kV six-phase operation does offer some reduced radio interference and audible noise compared with uprating to 230-kV double-circuit three-phase operation.

Gross and Thompson [37] have presented a method for calculating phase and sequence voltage and currents throughout a power system of mixed phase order subjected to an arbitrary fault type at an arbitrary bus in the system, and have described the procedure on a power system network having twelve-phase transmission line.

The fault analysis on a twelve-phase transmission line has been carried out by Pal and Singh [38] , and 23 types of faults are analysed with the help of symmetrical component transformation for the twelve-phase system. From the study of the 12-phase to ground fault and the single-line to ground fault, it has been shown that the fault level for a twelve-

phase system is less than that for a three-phase system. It is observed that, the faults other than the twelve-phase to ground, are dual in pair, for example, single-phase to ground and eleven-phase to ground, as the sequence networks are connected in parallel for eleven-phase to ground fault while they are connected in series in the case of single-phase to ground fault, similarly ten-phase to ground fault and two-phase to ground fault and so on.

Swarup et al [39] have carried out the load flow investigation on a combined three-phase and twelve-phase transmission lines under balanced as well as under unbalanced condition by deriving the models of power system elements required for such studies.

Tiwari and Singh [40] have carried out the transient stability analysis on a mixed three-phase and six-phase system, and have investigated the impact of converting the double-circuit three-phase lines to single-circuit six-phase lines with the same adjacent phase voltage, and therefrom the mixed transmission system has been shown to be better than the completely three-phase system from stability point of view.

Saleh and Laughton [41] have developed mathematical representations for the various three-phase/six-phase transformers in the phasor co-ordinates.

The efforts towards the design and fabrication for the larger rating of a synchronous machine have been made since a long time [42-44]. But the attempt was initially hampered by certain limitations in the circuit breaker interrupting capacity and the enormous reactors needed to limit the fault current. To alleviate the difficulty, a double-winding generator was suggested [43] in which each of the two separate stator windings is connected to a different sections of the station bus; the generated voltage in the two windings are equal in magnitude and are in phase. Several types of windings have been considered [44] . These machines have been found adequate until the fault current problem is alleviated by connecting each generator to its own step-up transformer. However, the problem of enlarging the generator rating accrues from the limitations arising out of stator voltage, mechanical forces on the stator coils, stray-surface losses and the physical strength of the rotor forging and the major coil retaining rings.

The problem of the coil forces has been solved with the ordinary double-winding [45] . However, the unsymmetrical winding causes additional losses and create difficulties in the end winding arrangement. To avoid these problems, two separate stator windings displaced at 30 electrical degrees have been suggested [46] .

P. Robert et al [46] have shown that the stray losses due to phase harmonics can be materially reduced by means of two separate stator windings displaced at 30 electrical degrees. The methods of protection of multi-phase generator against faults have also been dealt with [47] .

Holley et al [48] have shown that electromagnetic forces on the stator windings of large turbine generators can be substantially reduced by using the winding arrangements (two three-phase stator windings displaced at 30 electrical degrees) that permit an increased number of stator slots. This generator has been shown to have a very low harmonic contents. It has been found to be particularly attractive for feeding power to a dc transmission grid because of their reduced sensitivity to harmonics in the line current that might be introduced by the rectifier system.

Notwithstanding the great strides being made, two-pole generators appear to be reaching a plateau at about 1500 MVA [49] . Fuchs and Rosenberg [50] have carried out the steady-state analysis of a six-phase synchronous generator by conceptualizing it as the combination of two three-phase stator windings with 30 degree displacement by means of an orthogonal transformation which has been taken as a set of two three-phase Park's transformation in dqo variables.

The use of a six-phase synchronous generator along with its associated six-phase/three-phase transformers for

high power applications have been discussed by Hanna et al [51].

The six-phase synchronous machine has been a alysed by Schiferl and Ong in their companion papers [52,53]. In their first paper [52] a harmonic phasor method for steady-state analysis of a six-phase machine with mixed AC-DC stator connection has been examined, and in their second paper [53] , a novel application of the six-phase machine in an uninterruptible power supply scheme has been proposed.

Willems [54] has indicated that the six-phase generator and its associated transformers can be treated for fault analysis employing symmetrical component transformation. Further, a three-phase equivalent representation has been obtained by combining the generator voltage and the impedances with the equivalent circuit of the transformer.

After this brief literature survey, a summary of the work carried out in this thesis and its chapterwise discription are presented.

1.3 OBJECTIVE AND SCOPE OF WORK REPORTED IN THE THESIS

The objectives of this thesis are set forth as follows :

1. to develop a dynamic model of a six-phase synchronous machine in order to carry out the dynamic stability analysis of the synchronous machine and also to compare the

stability performance of a six-phase machine to that of a conventional three-phase synchronous machine;

2. to develop suitable mathematical models of various multi-phase elements of power system networks, viz., synchronous machines, transformers and transmission lines for a general n-phase (order) system under steady-state conditions;

3. to carryout the transient stability investigations employing the mathematical models in order to study the multi-phase system especially the impact of converting double-circuit three-phase lines to single circuit multi-phase lines;

4. to study the dynamic stability on a combined three-phase and multi-phase transmission lines with state-space formulation; and

5. to study the performance and feasibility of six-phase systems with the aid of several case studies.

1.4 CHAPTERWISE DESCRIPTIONS

A chapterwise description of the work carried out in the thesis is presented as follows.

Chapter 2 starts with the inductance calculations of the six-phase synchronous machine from the fundamentals. The dynamical equations of the machine have been developed with the help of a generalized park's transformation. Further,

a linearized model of the machine has been obtained in terms of current variables. The six-phase (multi-phase) synchronous machine has been analysed for the different conditions, and the performances of the machine has been compared to those of the three-phase machine from the point of dynamic stability.

Chapter 3 presents generalized mathematical models of the various components of n-phase power system elements under steady-state condition. A mathematical model of multi-phase synchronous machine in generalized form has been derived, and from which that of the six-phase machine has been obtained. A three-phase/n-phase transformer will be required when n-phase line is connected to a three-phase line and therefore, it requires a model for such transformers. Therefore, the models for three-phase/n-phase transformers with tappings for various connections, such as three-phase star/n-phase star, three-phase delta/n-phase star, etc., have been developed. The three-phase equivalent of a combined n-phase transmission line and three-phase/n-phase transformers have been developed in terms of ABCD parameters representing π-circuit. The n-phase equivalent of a three-phase transmission line with an appropriate transformer at each end of the line has been obtained. These equivalent representations are required to carry out the analysis of the overall system entirely either on 3-phase or on n-phase basis. Further, a single-phase equivalent of the n-phase transmission line with an appropriate transformer at the each end of the

line has been derived from its three-phase equivalent in terms of ABCD parameters representing π-circuit. This representation is adequate to analyse the entire system, if it is balanced and symmetric, on a single-phase basis.

In chapter 4, the load flow investigation has been carried out on a mixed three-phase and six-phase lines and also on a mixed three-phase twelve-phase lines to study the performance of the mixed power system in comparision to that of a completely three-phase system for the various configuration. To study the effect of conversion of double-circuit three-phase lines to single circuit multi-phase (six-phase or twelve-phase) lines, two voltage conditions have been assumed, namely, the phase to ground voltage of a multi-phase line is equal or $\sqrt{3}$ times that of a double circuit three-phase line. Two illustrative examples have been presented for the sake of comparision.

Chapter 5 deals with the transient stability investigation of the multi-phase systems. The direct axis and the quadrature axis transient and subtransient inductances of a six-phase synchronous machine have been obtained. The various time constants pertaining to the synchronous machine have also been determined. Further, the transient stability investigation on a sample network has been carried out for the different system configurations, for example, a completely three-phase system with a few double circuit three-phase line, a combined

three-phase and six-phase lines where these multi-phase lines are obtained by converting double circuit three-phase lines, and a completely six-phase (multi-phase) systems. The critical clearing lines have been calculated for the different fault locations. The performance of multi-phase lines with respect to double-circuit three-phase lines have been examined.

Chapter 6 presents the dynamic stability analysis of the power system with six-phase transmission lines. The dynamic model of a power system has been developed in terms of voltage and current variables and the dynamic stability has been investigated with the help of eigenvalue techniques for the different system configurations on a sample network. The dynamic stability performance due to the uprating of double-circuit three-phase lines to single circuit multi-phase lines have been investigated.

Finally, chapter 7 concludes with the overview of the work reported in this thesis and further, outlines briefly the future scope of work in this area.

1.5 CONCLUDING REMARKS

This chapter presents a brief account of motivation for taking up the present work and further, depicts the work previously carried out in the area of multi-phase systems. The objective and scope of the thesis have been explained briefly and the chapterwise work reported in this thesis has been summarized.

CHAPTER 2

DYNAMICAL MODELLING AND ANALYSIS OF A MULTI-PHASE (SIX-PHASE) SYNCHRONOUS MACHINE

2.1 INTRODUCTION

In the previous chapter, the work reported in the thesis has been summarized along with the literature survey on multi-phase power systems. This chapter presents a detailed dynamic model of a six-phase (multi-phase) synchronous machine, and also its analysis. As it is intended to carry out the dynamic stability investigation of a six-phase synchronous machine, a mathematical model of the machine under dynamic condition is required to be developed. Such models for three-phase synchronous machines are available in literature [55-59], but for six-phase machines this is almost untouched.

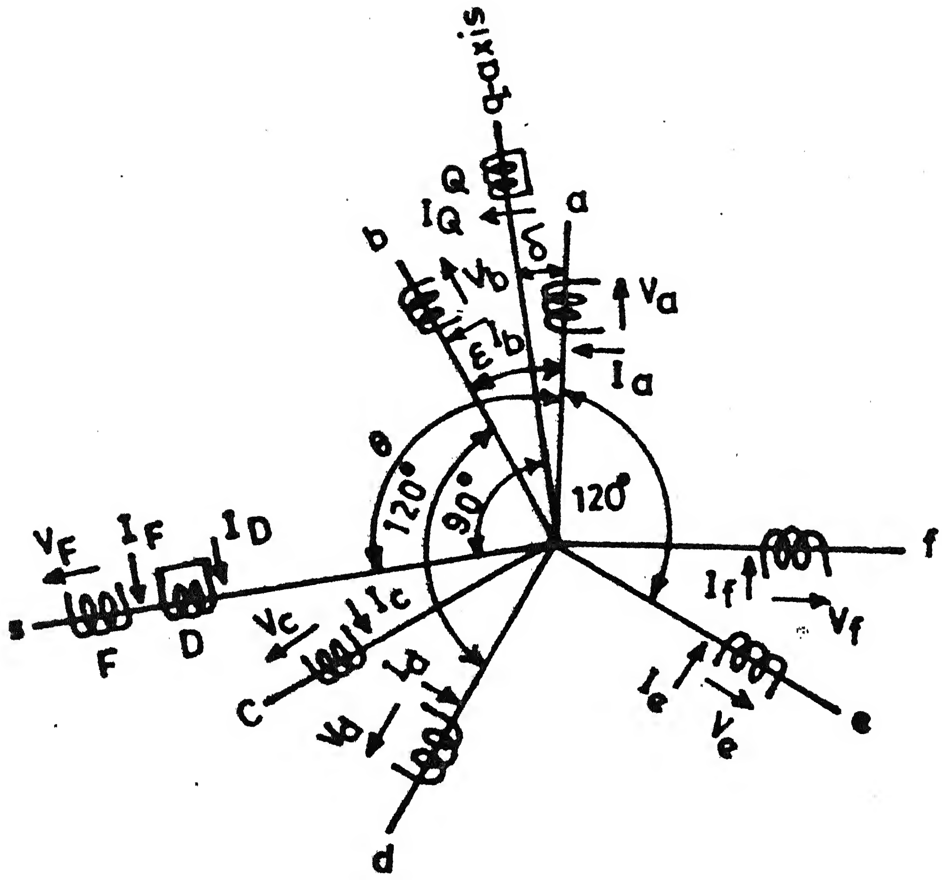
This chapter initiates in section 2.2.1 with the calculation of the inductances of the six-phase (multi-phase) machine from the fundamentals. Further, dynamical equations of the machine have been derived in section 2.2.3 with the help of on orthogonal power invariant transformation presented in section 2.2.2.

In order to investigate the dynamic stability of the six-phase (multi-phase) machine, a linearized model in state space form has been derived in section 2.3. Further, in section 2.4 a case study has been carried out on a sample sys-

tem to investigate the dynamic stability of the six-phase machine employing eigenvalue technique. In section 2.4.1, methods for finding the initial conditions of a six-phase synchronous machine have been provided for two boundary conditions. Then, with the help of linearized model developed in section 2.3 and the initial conditions obtained for two boundary conditions from the machine parameters, the dynamic stability of a six-phase machine with and without damper windings has been investigated in section 2.4.2 and 2.4.3 respectively. The results of the investigation have been presented and discussed in section 2.4.4 and finally, the chapter concludes in section 2.5 with the finding of the investigation.

2.2 DYNAMICAL MODELLING OF A SIX-PHASE (MULTI-PHASE) SYNCHRONOUS MACHINE

In order to develop a detailed dynamic model, a six-phase synchronous machine has been considered to consist of six-windings (two sets of balanced three-phase winding displaced at ε electrical degrees) on the stator, one field winding F on the rotor d-axis, and two fictitious short circuited windings on the rotor d and q-axes to represent damping effect as shown in Fig. 2.1. One additional (short circuited) winding 'G' on the rotor q-axis (not shown in the Fig. 2.1) has been included to take eddy current effects into consideration. In this section, inductances of the machine windings have been



5.2.1 Schematic winding arrangement of a Multi-phase (six-phase) synchronous machine.

calculated from the fundamentals, and further the dynamic model has been derived with the help of an orthogonal transformation.

2.2.1 Inductances of a Six-Phase (multi-phase) Synchronous Machine

The machine has been considered to consist of six windings namely a,b,c,d,e and f on the stator, one main field winding F on the rotor d-axis, the fictitious G winding on the q-axis, and two fictitious short-circuited D and Q windings on the rotor d- and q-axis respectively.

Stator Self-inductances:

Let us consider the flux linkages with any phase i of the stator winding. If only phase i is excited, the peak amplitude of the mmf wave due to the current in phase i is

$$F_i = N_i i_i \tag{2.1}$$

where N_i is the effective turns/phase and i_i is the current in phase i.

If F_{di} and F_{qi} are the d-axis and the q-axis components of the mmf, their amplitudes can be given as

$$F_{di} = F_i \cos\ (\theta - \theta_i) \tag{2.2}$$

$$\text{and}\quad F_{qi} = F_i\ \sin\ (\theta - \theta_i) \tag{2.3}$$

where θ is the angle between the reference phase and the rotor d-axis;

θ_i between the reference phase and the phase i in consideration.

If ρ_{gd} and ρ_{gq} be the permeances along the d-axis and the q-axis respectively, the fundamental air gap flux per pole along the two axes can be given as

$$\phi_{gdi} = \text{component of the air gap flux along the d-axis}$$

$$= F_{di} \cdot \rho_{gd} = F_i \rho_{gd} \cos(\theta - \theta_i) \qquad (2.4)$$

and ϕ_{gqi} = component of the air gap flux along the q-axis

$$= F_{qi} \rho_{gq} = F_i \rho_{gq} \sin(\theta - \theta_i) \qquad (2.5)$$

If ϕ_{gii} is the flux linking with the winding of the phase i when only phase i is excited, then

$$\phi_{gii} = \phi_{gdi} \cos(\theta - \theta_i) + \phi_{gqi} \sin(\theta - \theta_i) \qquad (2.6)$$

From equations (2.4) - (2.6), and using trigonometric identities

$$\phi_{gii} = N_i\, i_i \left[\frac{\rho_{gd} + \rho_{gq}}{2} + \frac{\rho_{gd} - \rho_{gq}}{2} \cos 2(\theta - \theta_i) \right] \qquad (2.7)$$

Hence,

$$L_{gii} = N_i\, \phi_{gii}/i_i = L'_{aa0} + L_{aa2} \cos 2\,(\theta - \theta_i) \qquad (2.8)$$

where

$$L'_{aa0} = N_i^2\,(\rho_{gd}+\rho_{gq})/2\ ;\ L_{aa2} = N_i^2(\rho_{gd}-\rho_{gq})/2 \qquad (2.9)$$

If the flux which does not cross the air gap (i.e. leakage flux) is also considered, the self inductance L_{ii} can be expressed as

$$L_{ii} = L_{li} + L_{gii} = L_{aa0} + L_{aa2} \cos 2\,(\theta - \theta_i) \qquad (2.10)$$

Equation (2.10) thus gives in general the self-inductance of the stator winding i (for i = a,b,c,d,e,f).

If the six windings on the stator are considered as two sets of balanced three-phase winding displaced at ε electrical degrees (where $\varepsilon = 60^\circ$ for six-phase machine), the self inductances of the winding can be computed from equation (2.10), and are as follows (assuming the axis of phase a to be the reference axis)

$$L_{aa} = L_{aa0} + L_{aa2} \cos 2\theta \qquad (2.11)$$

$$L_{bb} = L_{aa0} + L_{aa2} \cos\,(2\theta - 2\varepsilon) \qquad (2.12)$$

$$L_{cc} = L_{aa0} + L_{aa2} \cos\,(2\theta - 240^\circ) \qquad (2.13)$$

$$L_{dd} = L_{aa0} + L_{aa2} \cos\,(2\theta - 240^\circ - 2\varepsilon) \qquad (2.14)$$

$$L_{ee} = L_{aa0} + L_{aa2} \cos (2\theta + 240^{\circ}) \tag{2.15}$$

$$L_{ff} = L_{aa0} + L_{aa2} \cos (2\theta + 240^{\circ} - 2\varepsilon) \tag{2.16}$$

Stator windings mutual inductances :

Let the mutual inductance between the phase i and the phase j of the stator windings be L_{ij}. Let ϕ_{gij} be the air gap flux linking with phase j when only phase i is excited. The air gap flux is given by

$$\phi_{gij} = \phi_{gdi} \cos (\theta - \theta_j) + \phi_{gqi} \sin (\theta - \theta_j) \tag{2.17}$$

where θ_j is the angle between the flux linking phase j and the reference phase.

Substituting the values of ϕ_{gdi} and ϕ_{gqi} from equations (2.4) and (2.5) in equation (2.17), we have,

$$\phi_{gij} = N_i I_i \left[\frac{\rho_{gd} + \rho_{gq}}{2} \cos (\theta_i - \theta_j) + \frac{\rho_{gd} - \rho_{gq}}{2} \cos (2\theta - \theta_i - \theta_j) \right] \tag{2.18}$$

Then, the mutual inductance between the phases i and j due to the air gap flux can be given by

$$L_{ij} = L_{abo} \cos (\theta_i - \theta_j) + L_{aa2} \cos (2\theta - \theta_i - \theta_j) \tag{2.19}$$

where $L_{abo} = L'_{aa0}$ in equation (2.9).

The equation (2.19) gives the mutual inductance between the phases i and j and therefore, the mutual inductances between the stator windings can be obtained and are given below by

using axis of phase a as the reference.

$$L_{ab} = L_{abo} \cos \varepsilon + L_{aa2} \cos (2\theta - \varepsilon) \quad (2.20a)$$

$$L_{ac} = L_{abo} \cos 120^{\circ} + L_{aa2} \cos (2\theta - 120^{\circ}) \quad (2.20b)$$

$$L_{ad} = L_{abo} \cos (-120^{\circ} - \varepsilon) + L_{aa2} \cos (2\theta - 120^{\circ} - \varepsilon) \quad (2.20c)$$

$$L_{ae} = L_{abo} \cos 120^{\circ} + L_{aa2} \cos (2\theta + 120^{\circ}) \quad (2.20d)$$

$$L_{af} = L_{abo} \cos (120^{\circ} - \varepsilon) + L_{aa2} \cos (2\theta + 120^{\circ} - \varepsilon) \quad (2.20e)$$

$$L_{bc} = L_{abo} \cos (-120^{\circ} + \varepsilon) + L_{aa2} \cos (2\theta - 120^{\circ} - \varepsilon) \quad (2.20f)$$

$$L_{bd} = L_{abo} \cos (-120^{\circ}) + L_{aa2} \cos (2\theta - 120^{\circ} - 2\varepsilon) \quad (2.20g)$$

$$L_{be} = L_{abo} \cos (120^{\circ} + \varepsilon) + L_{aa2} \cos (2\theta + 120^{\circ} - \varepsilon) \quad (2.20h)$$

$$L_{bf} = L_{abo} \cos 120^{\circ} + L_{aa2} \cos (2\theta + 120^{\circ} - 2\varepsilon) \quad (2.20i)$$

$$L_{cd} = L_{abo} \cos \varepsilon + L_{aa2} \cos (2\theta - 240^{\circ} - \varepsilon) \quad (2.20j)$$

$$L_{ce} = L_{abo} \cos 240^{\circ} + L_{aa2} \cos 2\theta \quad (2.20k)$$

$$L_{cf} = L_{abo} \cos (240^{\circ} - \varepsilon) + L_{aa2} \cos (2\theta - \varepsilon) \quad (2.20l)$$

$$L_{de} = L_{abo} \cos (240^{\circ} + \varepsilon) + L_{aa2} \cos (2\theta - \varepsilon) \quad (2.20m)$$

$$L_{df} = L_{abo} \cos 240^{\circ} + L_{aa2} \cos (2\theta - 2\varepsilon) \quad (2.20n)$$

$$L_{ef} = L_{abo} \cos \varepsilon + L_{aa2} \cos (2\theta + 240^{\circ} - \varepsilon) \quad (2.20p)$$

Stator to Rotor Mutual Inductances :

These inductances vary with the position of the rotor, since it has salient pole construction.

The mutual inductance between any phase i of the stator winding and the rotor winding can be expressed as

$$L_{Fi} = L_{iF} = L_{mF} \cos (\theta - \theta_i) \quad (2.21)$$

$$L_{Di} = L_{iD} = L_{mD} \cos (\theta - \theta_i) \quad (2.22)$$

$$L_{Qi} = L_{iQ} = - L_{mQ} \sin (\theta - \theta_i) \tag{2.23}$$

$$L_{Gi} = L_{iG} = - L_{mG} \sin (\theta - \theta_i) \tag{2.24}$$

where $i = a,b,c,d,e,f.$

Rotor Inductances :

The self inductances of the rotor windings, namely L_{FF}, L_{DD}, L_{QQ}, L_{GG} are constant, as these values do not depend on the rotor position because the armature has cylindrical structure (only rotor has salient pole construction).

The mutual inductances between the rotor windings are also constant. L_{FD} and L_{QG} are non-zero while L_{DQ}, L_{FQ}, L_{GD} and L_{GF} are zero as the corresponding windings (referred to by the capital letter subscripts) are at right angles to each other.

2.2.2 Generalized Orthogonal Transformation

In order to develop the dynamic model **an** orthogonal transformation for a n-phase machine is required, as the self and the mutual inductances of the machine derived in section 2.2.1 are time dependent. Such type of orthogonal transformation known as Park's transformation for three-phase system is available in the literature [55-56]. For six-phase system, Fuchs and Rosenberg [50] have taken orthogonal transformation as the two sets of three-phase transformation. Here, the orthogonal transformation for the n-phase system is presented

and which is as follows :

$$T_{pn} = \sqrt{\frac{2}{n}} \begin{bmatrix} \cos\theta & \cos(\theta-m) & \cos(\theta-2m) & \cdots & \cos[\theta-(n-1)m] \\ \sin\theta & \sin(\theta-m) & \sin(\theta-2m) & \cdots & \sin[\theta-(n-1)m] \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (2.2$$

where n is the number of phases and $m = 2\pi/n$.

This transformation, which can be considered as a generalized Park's transformation for n-phase system, transforms the phasor time varying quantities into component time-invariant quantities and thus simplifies the analysis.

It can be seen that this generalized transformation is linear and power-invariant. This is in general a rectangular matrix and its Pseudo-inverse is given by

$$T_{pn}^{-1} = T_{pn}^{t} \left[T_{pn} \times T_{pn}^{t} \right]^{-1} \quad (2.26)$$

where, t denotes the transpose of the matrix.

Thus, the Pseudo-inverse of T_{pn} is

$$T_{pn}^{-1} = \sqrt{\frac{2}{n}} \begin{bmatrix} \cos\theta & \sin\theta & 1/\sqrt{2} \\ \cos(\theta-m) & \sin(\theta-m) & 1/\sqrt{2} \\ \vdots & \vdots & \vdots \\ \cos[\theta-(n-1)m] & \sin[\theta-(n-1)m] & 1/\sqrt{2} \end{bmatrix} \quad (2.27)$$

If the six-phase winding of the stator is constructed as the two sets of balanced three-phase windings displaced at ε

electrical degree, the corresponding transformation can be given by

$$T_{p6} = \sqrt{\frac{2}{6}} \begin{bmatrix} \cos\theta & \cos(\theta-\varepsilon) & \cos(\theta-120^\circ) & \cos(\theta-120^\circ-\varepsilon) & \cos(\theta+120^\circ) & \cos(\theta+120^\circ-\varepsilon) \\ \sin\theta & \sin(\theta-\varepsilon) & \sin(\theta-120^\circ) & \sin(\theta-120^\circ-\varepsilon) & \sin(\theta+120^\circ) & \sin(\theta-120^\circ-\varepsilon) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \qquad (2.28)$$

By substituting corresponding ε, a linear orthogonal power invariant transformation can be obtained for the purpose of simplifying the analysis.

As the rotor quantities are not affected with this transformation, the augmented transformation T'_{P6} can be obtained by including also rotor windings and the transformation T'_{P6} thus becomes,

$$T'_{P6} = \begin{bmatrix} T_{P6} & 0 \\ 0 & I \end{bmatrix} \qquad (2.29)$$

where I is the identity matrix of order (4 x 4)

2.2.3 Dynamical Equations of a Six-Phase Machine

The voltage equations for the six-phase synchronous machine in the phasor form is given by

$$\underline{V}_P = -[r]\,\underline{i}_P - \frac{d}{dt}\underline{\Psi}_P \qquad (2.30)$$

where,

$$\underline{V}_P = [V_a\ V_b\ \ldots\ V_f\ V_F\ V_G\ V_D\ V_Q]^t \qquad (2.31a)$$

$$\underline{i}_P = [i_a\ i_b\ \ldots\ i_f\ i_F\ i_G\ i_D\ i_Q]^t \qquad (2.31b)$$

$$\underline{\Psi}_P = [\Psi_a\ \Psi_b\ \ldots\ \Psi_f\ \Psi_F\ \Psi_G\ \Psi_D\ \Psi_Q]^t \qquad (2.31c)$$

and $[r]$ is a diagonal matrix whose diagonal elements are the resistances of the windings, and $V_G = V_D = V_Q = 0$ as these windings are short-circuited.

The flux linkages of the windings in the phasor form is

$$\underline{\Psi}_P = [L_{a..Q}]\,\underline{i}_P \qquad (2.32)$$

where $[L_{a..Q}]$ is the inductance coefficient matrix and is given as

$$[L_{a..Q}] = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & \cdot\cdot & L_{af} & L_{aF} & L_{aG} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & \cdot\cdot & L_{bf} & L_{bF} & L_{bG} & L_{bD} & L_{bQ} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{Qa} & L_{Qb} & L_{Qc} & \cdot\cdot & L_{Qf} & L_{QF} & L_{QG} & L_{QD} & L_{QQ} \end{bmatrix} \qquad (2.33)$$

The current, flux and the voltage in the transformed components are,

$$\underline{i}_{park} = [T'_{P6}]\ \underline{i}_P \tag{2.34}$$

$$\underline{\Psi}_{Park} = [T'_{P6}]\ \underline{\Psi}_P \tag{2.35}$$

$$\underline{v}_{Park} = [T'_{P6}]\ \underline{v}_P \tag{2.36}$$

Multiplying (2.32) by $[T'_{P6}]$ and substituting the value from equations (2.34) and (2.35), in following expression is obtained.

$$\underline{\Psi}_{Park} = [T'_{P6}][L_{a..Q}][T'_{P6}]^t\ \underline{i}_{Park} \tag{2.37}$$

Substituting the values of $[L_{a..Q}]$ from equations (2.11)-(2.16), (2.20a-2.20p) and (2.21- 2.24) in equation (2.37), we have,

$$\begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_o \\ \Psi_F \\ \Psi_G \\ \Psi_D \\ \Psi_Q \end{bmatrix} = \begin{bmatrix} L_d & 0 & 0 & \sqrt{3}\,L_{mF} & 0 & \sqrt{3}\,L_{mD} & 0 \\ 0 & L_q & 0 & 0 & -\sqrt{3}\,L_{mG} & 0 & -\sqrt{3}\,L_{mQ} \\ 0 & 0 & L_o & 0 & 0 & 0 & 0 \\ \sqrt{3}\,L_{mF} & 0 & 0 & L_{FF} & 0 & L_{FD} & 0 \\ 0 & -\sqrt{3}L_{mG} & 0 & 0 & L_{GG} & 0 & -L_{GQ} \\ \sqrt{3}L_{mD} & 0 & 0 & L_{FD} & 0 & L_{DD} & 0 \\ 0 & -\sqrt{3}L_{mQ} & 0 & 0 & -L_{GQ} & 0 & L_{QQ} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_o \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix} \quad (2.3$$

where,

$$L_d = L_{aa0} + 2L_{abo} + 3L_{aa2} \tag{2.39a}$$

$$L_q = L_{aa0} + 2L_{abo} - 3L_{aa2} \tag{2.39b}$$

$$L_o = L_{aa0} - L_{abo} \tag{2.39c}$$

Further, the equation (2.30) is manipulated with the help of equations (2.34)-(2.36) to obtain the voltage equations in the transformed variables and these are given in the matrix form as below.

$$\begin{bmatrix} v_d \\ v_q \\ v_o \\ v_F \\ v_G=0 \\ v_D=0 \\ v_Q=0 \end{bmatrix} = - \begin{bmatrix} r_a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_a & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_o \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix} - \begin{bmatrix} P\Psi_d \\ P\Psi_q \\ P\Psi_o \\ P\Psi_F \\ P\Psi_G \\ P\Psi_D \\ P\Psi_Q \end{bmatrix} + \begin{bmatrix} -\omega\Psi_q \\ \omega\Psi_d \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (2.40)$$

where, $P = d/dt$ and $\omega = d\theta/dt$.

Thus, equations (2.40) in matrix form gives the voltage equations in d,q,o component for a six-phase machine

Equation of Motion

The dynamic equations corresponding to the electromechanical motion of the machine are

$$\frac{d\delta}{dt} = \omega - \omega_o \qquad (2.41)$$

and $$M \frac{d\omega}{dt} = [T_m - T_e - K_D (\omega - \omega_o)] \qquad (2.42)$$

where M is inertia constant in sec/radian/sec.

T_m is the mechanical torque

T_e is the electrical torque

K_D is the damping coefficient.

2.3 LINEARIZED MODEL OF A SIX-PHASE (MULTI-PHASE) SYNCHRONOUS MACHINE

A detailed linear model of a six-phase (multi-phase) synchronous machine in the state-space form has been developed using the currents as state-varaibles. The machine has been considered to consist of six windings on the stator, one field winding F and two fictitious windings D and Q on the rotor. The performance equations to represent the six-phase synchronous machine have been derived on the same lines as those in section 2.2. All the quantities are in p.u. except time and angle which are in second and radiant respectively.

$$\Psi_d = X_d i_d + \sqrt{3}\, X_{mF}\, i_F + \sqrt{3}\, X_{mD}\, i_D \tag{2.43}$$

$$\Psi_F = \sqrt{3} X_{mF}\, i_d + X_{FF}\, i_F + X_{FD}\, i_D \tag{2.44}$$

$$\Psi_D = \sqrt{3} X_{mD}\, i_d + X_{FD}\, i_F + X_{DD}\, i_D \tag{2.45}$$

$$\Psi_q = X_q\, i_q - \sqrt{3}\, X_{mQ}\, i_Q \tag{2.46}$$

$$\Psi_Q = -\sqrt{3}\, X_{mQ}\, i_q + X_{QQ}\, i_Q \tag{2.47}$$

$$v_d = -r_a\, i_d - \frac{1}{\omega_o} P\Psi_d - \frac{\omega}{\omega_o} \Psi_q \tag{2.48}$$

$$v_F = -r_F\, i_F - \frac{1}{\omega_o} P\,\Psi_F \tag{2.49}$$

$$v_D = 0 = -\, r_D\, i_D - \frac{1}{\omega_o} P\Psi_D \tag{2.50}$$

$$v_q = -r_a\, i_q - \frac{1}{\omega_o} P\,\Psi_q + \frac{\omega}{\omega_o} \Psi_d \tag{2.51}$$

$$v_Q = 0 = -\, r_Q\, i_Q - \frac{1}{\omega_o} P\,\Psi_Q \tag{2.52}$$

$$P\delta = \omega - \omega_o \tag{2.53}$$

$$P\omega = \frac{1}{M}\left[T_m - T_e - K_D\,(\omega - \omega_o) \right] \tag{2.54}$$

$$T_e = \Psi_d\, i_q - \Psi_q\, i_d \tag{2.55}$$

The set of equations (2.43)-(2.55) have been manipulated to obtain the state space model as follows.

$$\underline{E}\dot{\underline{X}} = f(\underline{X}) + \underline{D}V \tag{2.56}$$

where

$$\underline{X} = [\,\delta,\ \omega,\ i_d,\ i_q,\ i_F,\ i_D,\ i_Q]^t \tag{2.57a}$$

$$V = T_m \tag{2.57b}$$

and

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_d & 0 & \sqrt{3}X_{mF} & \sqrt{3}X_{mD} & 0 \\ 0 & 0 & 0 & X_q & 0 & 0 & -\sqrt{3}X_{mQ} \\ 0 & 0 & \sqrt{3}X_{mF} & 0 & X_{FF} & X_{FD} & 0 \\ 0 & 0 & \sqrt{3}X_{mD} & 0 & X_{FD} & X_{DD} & 0 \\ 0 & 0 & 0 & -\sqrt{3}X_{mQ} & 0 & 0 & X_{QQ} \end{bmatrix} \tag{2.57c}$$

$$f(\underline{X}) = \begin{bmatrix} \omega - \omega_o \\ -\frac{K_D}{M}(\omega - \omega_o) - \frac{1}{M}(X_d\, i_q\, i_d + \sqrt{3}X_{mF}\, i_q\, i_F + \sqrt{3}X_{mD}\, i_q\, i_D - \\ \qquad X_q\, i_d\, i_q + \sqrt{3}X_{mQ} i_d i_Q) \\ -\omega_o r_a i_d - \omega X_q i_q + \sqrt{3}\,\omega X_{mQ}\, i_Q + \omega_o v_t\, \sin\delta \\ -\omega_o r_a i_q + \omega X_d i_d - \sqrt{3}\,\omega X_{mF} i_F + \sqrt{3}\,\omega X_{mD}\, i_D - \omega_o v_t\, \cos\delta \\ -\omega_o r_F i_F - (\omega_o r_F / X_{mF})\, E_{FD} \\ -\omega_o r_D i_D \\ -\omega_o r_Q i_Q \end{bmatrix} \tag{2.57d}$$

$$D = \left[0 \quad \frac{1}{M} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \right]^t \tag{2.57e}$$

Using Taylor's series expansion of equation (2.56) about an operating point and retaining the first order terms in the expansion, the linearized model is obtained in the form

$$E\dot{\underline{X}}_m = F\,\underline{X}_m + G\,\underline{u}_m \tag{2.58}$$

where,

$$\underline{X}_m = [\Delta\delta, \Delta\omega, \Delta i_d, \Delta i_q, \Delta i_F, \Delta i_D, \Delta i_Q]^t \tag{2.58a}$$

$$G = \begin{bmatrix} 0 & \frac{1}{M} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-\omega_o r_F}{X_{mF}} & 0 & 0 \end{bmatrix}^t \tag{2.58b}$$

$$\underline{u}_m = [\Delta T_m \quad \Delta E_{FD}]^t \tag{2.58c}$$

and

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{K_D}{M} & -\frac{a}{M} & -\frac{b}{M} & -\frac{\sqrt{3}X_{mF}i_{qo}}{M} & -\frac{\sqrt{3}X_{mD}i_{qo}}{M} & -\frac{\sqrt{3}X_{mQ}i_{do}}{M} \\ \omega_o v_t\cos\delta_o & -c & -\omega_o r_a & -\omega_o X_q & 0 & 0 & \sqrt{3}\omega_o X_{mQ} \\ \omega_o v_t\sin\delta_o & d & \omega_o X_d & -\omega_o r_a & \sqrt{3}\omega_o X_{mF} & \sqrt{3}\omega_o X_{mD} & 0 \\ 0 & 0 & 0 & 0 & -\omega_o r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\omega_o r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\omega_o r_Q \end{bmatrix}$$

(2.58d)

where

$$a = (X_d - X_q)\, i_{qo} + \sqrt{3}\, X_{mQ}\, i_{QO} \tag{2.58e}$$

$$b = (X_d - X_q)\, i_{do} + \sqrt{3}\, X_{mF}\, i_{FO} + \sqrt{3}\, X_{mD}\, i_{DO} \tag{2.58f}$$

$$c = X_q\, I_{qo} - \sqrt{3}\, X_{mQ}\, i_{Q0} \tag{2.58g}$$

$$d = X_d\, i_{do} + \sqrt{3}\, X_{mF}\, i_{FO} + \sqrt{3}\, X_{mD}\, i_{DO} \tag{2.58h}$$

The equation (2.58) reduces to

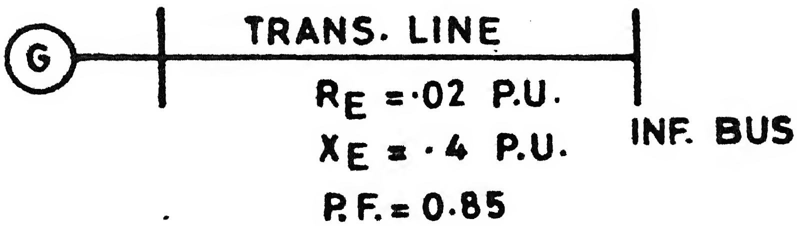
$$\dot{\underline{x}}_m = [A_m]\, \underline{x}_m + [B_m]\, \underline{u}_m \tag{2.59}$$

where $[A_m] = E^{-1} F$ and $[B_m] = E^{-1} G$. (2.60)

The equation (2.59) gives a detailed linearized model in state space form of a six-phase synchronous machine.

2.4 A CASE STUDY

The dynamic stability of a six-phase synchronous machine (Fig. 2.2) has been investigated first by taking damper windings D and Q into consideration and then by considering the machine without damper windings under two boundary conditions namely (a) the rated power and 0.85 lagging p.f. with generator terminal voltage as 1 pu and (b) the infinite bus voltage is 1.0 pu and the machine loading remains the same as in (a). The machine is connected by a transmission line of 0.02+j0.4 pu impedance to an infinite bus. The initial condition of the machines have been determined. The parameters of the six-phase synchronous machine have been obtained by assuming its induc-



IG.2.2 A six-phase (Multi-phase) machine connected to an Infinite bus through a transmission line.

tive coefficients equal to those of a three-phase synchronous machines, and the stability of six-phase and three-phase machines have been compared. However, the analysis of three-phase and six-phase machines have been carried out on pu values based on the single-phase quantities. The eigenvalues of system matrix $[A_m]$ have been determined to investigate the dynamic stability, and further, initial value responses of machine angle and angular velocity versus time have also been obtained.

2.4.1 Determination of Initial Conditions of Six-Phase Synchronous Machine

For the stability study, it is essential to determine the initial conditions prior to the application of the disturbance. Therefore, the steady state equation for six-phase machine is required for calculating the operating point conditions.

From equations (2.43) to (2.52), all the flux linkages and the currents at the steady-state are constant i.e.,

$$P\ \Psi_x = 0 \tag{2.61}$$

where $x = d, q, F, D, Q$

and also, $$i_D = i_Q = 0\ ;\quad \omega = \omega_o \tag{2.62}$$

Then, (2.48) and (2.45) give after manipulation

$$v_d = -r_a\ i_d - X_q\ i_q \tag{2.63}$$

and $\quad v_q = - r_a\, i_q + X_d\, i_d + \sqrt{3}\, X_{mF}\, i_F$ (2.64)

From equation (2.36), we have,

$$v_a = \sqrt{\tfrac{2}{6}}\,(v_d \cos\theta + v_q \sin\theta) \tag{2.65}$$

From equations (2.63)-(2.65) using $\theta = \omega_o t + \delta + \pi/2$

$$v_a = \sqrt{\tfrac{2}{6}}[-(r_a i_d + X_q i_q)\cos(\omega_o t+\delta+\pi/2) + (-r_a i_q + X_d i_d + \sqrt{3} X_{mF} i_F)\sin(\omega_o t+\delta+\pi/2)]$$

$$= \sqrt{\tfrac{2}{6}}[-(r_a i_d + X_q i_q)\cos(\omega_o t+\delta+\pi/2) + (-r_a i_q + X_d i_d + \sqrt{3} X_{mF} i_F)\cos(\omega_o t+\delta)]$$

-(2.66)

At open circuit, a field current i_F corresponds to an e.m.f. of $i_F\, X_{mF}$ peak and if the r.m.s. value of this peak is E, Then

$$i_F X_{mF} = \sqrt{2}E \quad \text{and} \quad \sqrt{3}\; i_F X_{mF} = \sqrt{6}\; E \tag{2.67}$$

when E is the stator equivalent EMF corresponding to i_F.

Introducing equation (2.67) in (2.66) by defining r.m.s. voltage phasor $\bar{V}_a = \dfrac{v_a}{\sqrt{2}}$, the following expression is obtained

$$\bar{V}_a = -r_a\left(\frac{i_q}{\sqrt{6}} \angle\delta + j\,\frac{i_d}{\sqrt{6}}\angle\delta\right) - j\,X_q \frac{i_q}{\sqrt{6}}\angle\delta + X_d \frac{i_d}{\sqrt{6}}\angle\delta + E\angle\delta \tag{2.68}$$

In this equation $\bar{V}_a$ and E are stator r.m.s. phase voltage in p.u.

If the r.m.s. equivalent d and q-axis currents are taken as

$$I_d \triangleq \frac{i_d}{\sqrt{6}} \quad \text{and} \quad I_q \triangleq \frac{i_q}{\sqrt{6}} \tag{2.69}$$

The stator current i_a, expressed as a phasor, will have two rectangular components I_q and I_d. Taking q-axis as the refe-

hence, we have,

$$\bar{I}_a = I_q + j\, I_d \qquad (2.70)$$

Substituting (2.69) and (2.70) in (2.68), we get,

$$E\underline{/\delta} = \bar{V}_a + r_a\bar{I}_a + jX_q\, I_q\underline{/\delta} - X_d\, I_d\, \underline{/\delta} \qquad (2.71)$$

By using $\bar{E} = E\underline{/\delta}$, $\bar{I}_q = I_q\underline{/\delta}$ and $\bar{I}_d = j\, I_d\underline{/\delta}$

Equation (2.71) becomes

$$\bar{E} = \bar{V}_a + r_a\bar{I}_a + j\, X_q\bar{I}_q + j\, X_d\, \bar{I}_d \qquad (2.72)$$

The equation (2.72) represents the steady state equation of six-phase machine and its phasor diagram is shown in Fig.2.3.

To obtain v_d and v_q from (2.63) and (2.64), the r.m.s. stator equivalent voltages can be computed as

$$V_d = \frac{v_d}{\sqrt{6}} = -\, r_a I_d - X_q\, I_q \qquad (2.73)$$

and $$V_q = \frac{v_q}{\sqrt{6}} = -\, r_a I_q + X_d I_d + E \qquad (2.74)$$

Case a : When the terminal voltage and the generated power of six-phase machine are known:

It is desired to obtain the initial condition of the six-phase machine under the above condition.

With the above boundary conditions, the terminal voltage v_a the current i_a and the power factor F_p are known.

Resolving $\bar{I}_a$ into components with $\bar{V}_a$ as the reference,

$$\bar{I}_a = I_r + j\, I_x \tag{2.75}$$

where I_r is the component of $\bar{I}_a$ in phase with $\bar{V}_a$ and I_x is its quadrature component, we also have,

$$F_p = \cos \phi_p$$

where ϕ_p is the angle by which $\bar{I}_a$ lags $\bar{V}_a$. Then,

$$I_r = I_a \cos \phi_p \; ; \; I_x = - I_a \sin \phi_p \tag{2.76}$$

The phasor $\bar{E}_{qa}$ in Fig. 2.4 is given by

$$\bar{E}_{qa} = V_a + (r_a + j\, X_q)\, \bar{I}_a = V_a + (I_r + jI_x)(r_a + jX_q)$$

$$= (V_a - x_q I_x + r_a I_r) + j\, (X_q I_r + r_a I_r) \tag{2.77}$$

The angle between the q-axis and the terminal voltage $\bar{V}_a$ is given by

$$\delta - \beta = \tan^{-1} [\, (x_q I_r + r_a I_x)/(V_a + r_a I_r - x_q I_x)\,] \tag{2.78}$$

Then we compute,

$$V_d = - V_a \sin (\delta - \beta) \; ; \; V_q = V_a \sin (\delta - \beta) \tag{2.79}$$

and v_d and v_q can then be determined from equations (2.73) and (2.74).

Further, the currents are obtained from

$$I_d = - I_a \sin (\delta - \beta + \phi) \; ; \; I_q = I_a \cos (\delta - \beta + \phi) \tag{2.80}$$

and then, i_d and i_q can be determined from relations in equation (2.69).

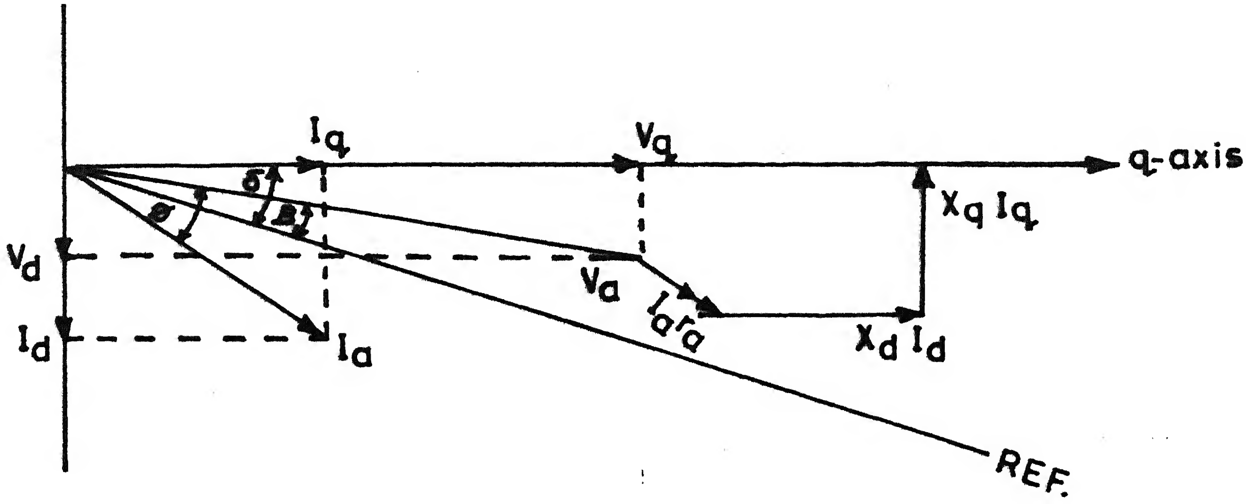


FIG.2.3 A phasor diagram of a Six-phase synchronous Machine.

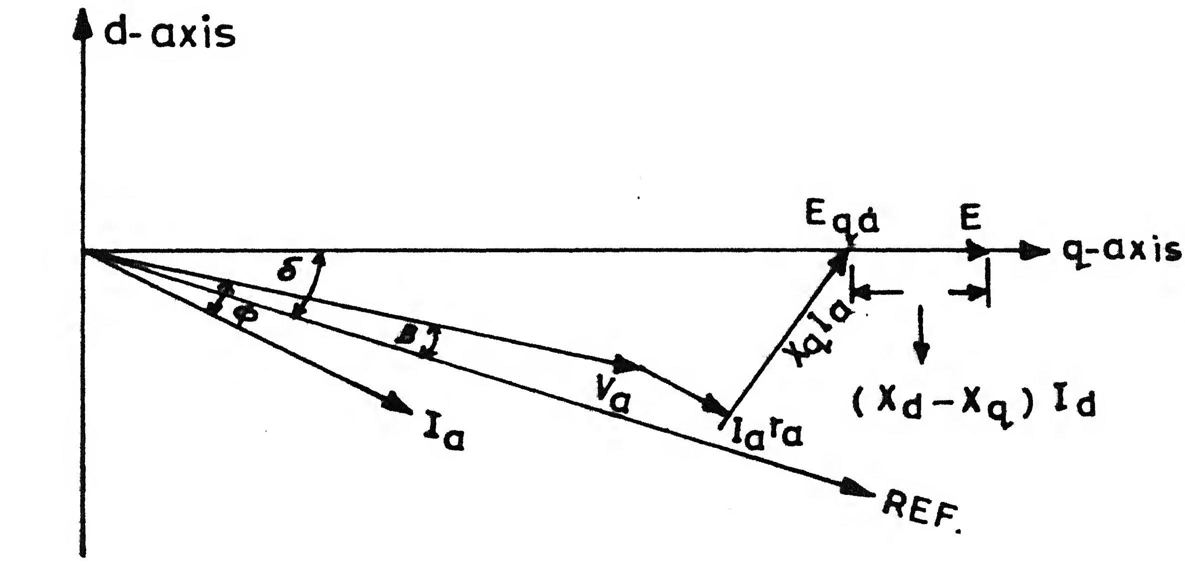


FIG.2.4 A phasor diagram of a Six-phase Machine for the location of q-axis under the boundary condition of Case a

From equation (2.74), E can be obtained.

$$\text{The field current } i_{Fo} = \sqrt{6}E/(\sqrt{3}\, X_{mF}) \qquad (2.81)$$

Thus, the initial conditions in terms of current variables can be known.

Case b : When the infinite bus voltage and the power of the six-phase machine at a particular power factor are known:

Since the boundary conditions in the case are mixed and therefore, the initial conditions can be obtained with the modification in (a) as shown below.

The machine is connected to the infinite bus through a transmission line having resistance and the reactance as R_e and X_e respectively.

The power at the infinite bus has been approximated on some estimate of current causing the power loss in the connecting lines.

The power delivered to the infinite bus = Power output of the

$$\text{machine} - I_a^2\, r_e \qquad (2.82)$$

The angle θ between $\bar{I}_a$ and V_∞ is given by

$$\tan\theta = \frac{I_x}{I_r} \qquad (2.83)$$

From Fig.2.5, the angle β between $\bar{V}_a$ and V_∞ is given by

$$\tan\beta = \frac{X_e I_r + R_e I_x}{V_\infty - X_e I_x + R_e I_r} \qquad (2.84)$$

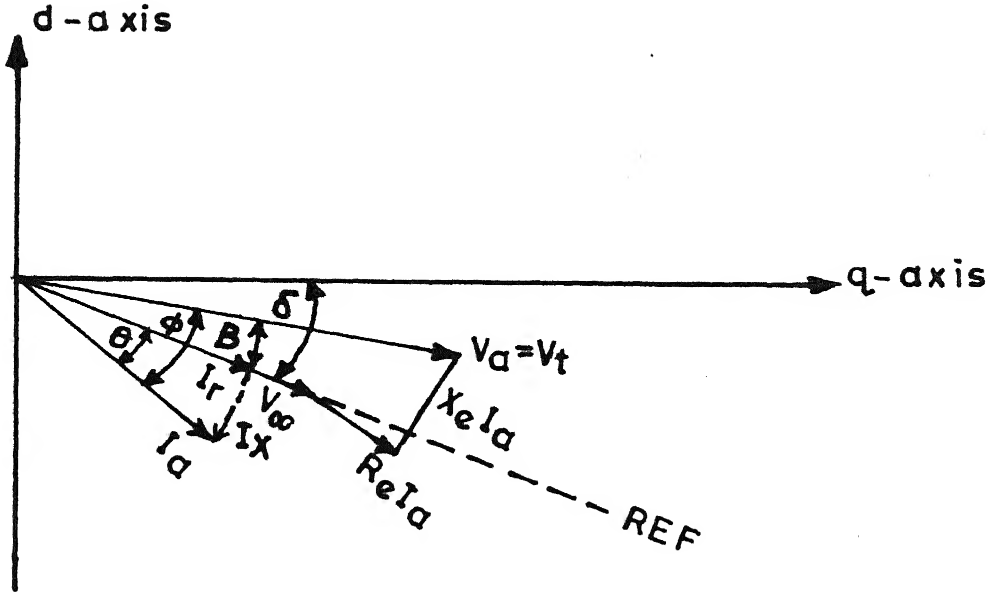


FIG. 2.5 A phasor diagram showing the terminal voltage of a six-phase machine and the voltage at the infinite bus.

The power factor angle ϕ at the machine terminal is given by

$$\phi = \beta + \theta \tag{2.85}$$

Using the trigonometric identify,

$$\tan \phi = (\tan \beta + \tan \Theta) \,/\, (1 - \tan \beta \tan \Theta) \tag{2.86}$$

From equations (2.85) and (2.86), I_x can be computed.

The terminal voltage V_a is given by

$$\bar{V}_a = (V_\infty - X_e I_x + R_e I_r) + j(X_e I_r + R_e I_x) \tag{2.87}$$

and

$$\tan \delta = [\,(X_q + X_e) I_r + (r_a + R_e) I_x\,] \,/\, [\,V_\infty - (X_q + X_e) I_x + (r_a + R_e) I_r\,] \tag{2.88}$$

Then the initial conditions of the synchronous machine in currents variables can be determined as given in case a.

2.4.2 Dynamic Stability of a Six-Phase Machine with Damper Windings.

A 60 Hz six-phase synchronous machine has been examined under two boundary conditions for which the methods for calculating the initial conditions has been discussed in the previous section. The machine is connected by a transmission line of (0.02+j0.4) pu impedance to an infinite bus. The parameters of the synchronous machine have been provided in Table 2.1. The six-phase machine parameters have been obtained by using those of the three-phase machine. Pu parameters have been provided on per phase basis. The parameters

X_d and X_q have been obtained from three-phase machine using equations (2.39a)-(2.39c) by assuming equal inductive co-efficients (L_{aa0}, L_{aa2}, L_{abo}) of the two machines. Data if any, not mentioned are referred to reference [55].

TABLE 2.1

Synchronous Machine Parameters

$X_d = 1.7$	$r_a = .001096$
$X_q = 1.64$	$r_F = .000742$
$^*X_d = 3.3$	$r_D = .0131$
$^*X_q = 3.18$	$r_Q = .054$
$X_{FF} = 1.65$	$\sqrt{\frac{3}{2}}\, X_{mQ} = 1.49$
$X_{DD} = 1.605$	$\sqrt{\frac{3}{2}}\, X_{mF} = X_{FD} = \sqrt{\frac{3}{2}}\, X_{mD} = 1.55$
$X_{QQ} = 1.526$	$M = .037719;\ K_D = 0.0$

* indicates the quantities which give the corresponding parameters for a six-phase synchronous machine

The initial conditions have been obtained as discussed in section 2.4.1 with the help of machine parameters, and have been provided in Table 2.2.

The six-phase machine has been considered under two conditions : 1) when its rated power is equal to the

rated power of a three-phase machine and 2) when its rated power is equal to twice that of the three-phase machine.

TABLE 2.2

Operating Conditions of Three-Phase and Six-Phase Machines Under Two Boundary Conditions

Boundary condition	quantity	3-Phase Machine rated Power	6-Phase Machine: rated Power equal to 3-Phase Machine	6-Phase Machine: rated Power equal to the twice of 3-Phase Machine
a	δ_o (in degree)	66.994	51.289	74.8296
	i_{do}	-1.9253	-1.3579	-2.8261
	i_{qo}	0.6673	0.4818	0.56369
	i_{FO}	2.9794	2.0626	3.5482
b	δ_o	53.7394	45.87	62.48
	i_{do}	-1.584	-1.132	-2.3695
	i_{qo}	0.6984	0.4676	0.6205
	i_{FO}	2.8194	1.914	3.1941

The dynamic stability has been investigated of the three-phase and six-phase machine under different conditions assuming both the machines with damper windings. This has been investigated by obtaining the eigenvalues of

the system matrix $[A_m]$ corresponding to the different operating conditions of three-phase and six-phase machines using program EIGRF of the IMSL subroutines. In this study, it is assumed that the input vector $\underline{u}_m$ i.e. $[\Delta T_m \quad \Delta E_{FD}]^t$ is zero. The eigenvalues of the system matrix $[A_m]$ are provided in Table 2.3.

TABLE 2.3

Eigenvalues of the System Matrix $[A_m]$ for Three-Phase and Six-Phase Machines with Damper Windings for Two Boundary Conditions

Boundary Condition	Eigenvalues	3-phase synchronous machine (rated power)	6-phase synchronous machine: equal power to 3-phase machine	6-phase synchronous machine: double power to 3-phase machine
a	λ_1, λ_2	$-13.54 \pm j376.335$	$-11.738 \pm j376.0$	$-11.74 \pm j376.0$
	λ_3, λ_4	$-0.3558 \pm j9.345$	$-0.8177 \pm j12.076$	$-0.5857 \pm j12.4157$
	λ_5, λ_6	$-37.71, -46.37$	$-39.81, -70.6854$	$-39.6974, -71.3896$
	λ_7	-0.20234	-0.3368	-0.212527
b	λ_1, λ_2	$-13.5513 \pm j376.335$	$-11.723 \pm j376.0$	$-11.742 \pm j376.0$
	λ_3, λ_4	$-0.5864 \pm j10.8792$	$-1.0675 \pm j12.5589$	$-0.8133 \pm j14.77$
	λ_5, λ_6	$-37.6865, -45.882$	$-39.8104, -70.194$	$-39.594, -70.96$
	λ_7	-0.2611	-0.3654	-0.286874

The machine angle and the angular velocity versus time responses have been obtained using IMSL subroutine DVERK

by applying a disturbance of $\Delta\delta = .1$ radian and these responses for case a and case b are provided from Figs. 2.6-2.9.

2.4.3 The Dynamic Stability of a Six-Phase Machine Without the Damper Windings

For the dynamic stability investigation of three-phase and six-phase machines without damper windings, the section 2.4.2 is repeated keeping in view that, when the linearized model in equation (2.59) is considered for the machines without damper windings, the rows and column corresponding to the damper winding quantities (6th and 7th rows and columns) are removed reducing the order of system matrix $[A_m]$ to (5x5), and correspondingly, matrix $[B_m]$ reduces to the order of (5x2). The initial conditions obtained in Table 2.2 can well be applied in this case for the different conditions of three-phase and six-phase machines, as the damper winding parameters do not affect the initial conditions of the machines.

The eigenvalues of the system matrix $[A_m]$ corresponding to three-phase and six-phase machines under two boundary condition have been provided in Table 2.4.

The machine angle and angular velocity response for three-phase and six-phase machines without damper wind-

ings under two boundary conditions with zero input vector have been shown in Figs. 2.10-2.13.

TABLE 2.4

Eigenvalues of the System Matrix $[A_m]$ for Three-Phase and Six-Phase Machines Without Damper Windings for Two Boundary Conditions

Boundary Condition	Eigenvalues	3-phase synchronous machine (rated power)	6-phase synchronous machine: equal power to the 3-phase machine	6-phase synchronous machine: double power to the 3-phase machine
a	λ_1, λ_2	$-8.129 \pm j376.9612$	$-6.1623 \pm j376.96$	$-6.1674 \pm j376.9624$
	λ_3, λ_4	$-0.165 \pm j9.254$	$-0.2173 \pm j11.8757$	$-0.277 \pm j12.22$
	λ_5	-0.2124	-0.352548	-0.2229
b	λ_1, λ_2	$-8.129 \pm j376.96$	$-6.154 \pm j376.957$	$-6.1658 \pm j376.962$
	λ_3, λ_4	$-0.135 \pm j10.7223$	$-0.2106 \pm j12.32$	$-0.239936 \pm j14.499$
	λ_5	-0.2738	-0.382365	-0.300374

2.4.4 Results and Discussions

The operating conditions of three-phase and six-phase synchronous machines under two boundary conditions, namely under case a and case b have been given in Table 2.2. Eigenvalues of system matrix $[A_m]$ corresponding to three-phase and six-phase synchronous machines with damper windings under two boundary conditions have been provided in

Table 2.3, and those corresponding to three-phase and six-phase machines without damper windings in Table 2.4. It is observed from Table 2.3 that the pair of eigenvalues (λ_3, λ_4) and the eigenvalue λ_7 of the system matrix $[A_m]$ almost determine the stability of the system. The pair λ_3, λ_4 in column 3 for three-phase machine corresponds to frequencies of approximately $(9.345/2\pi)$ 1.48 Hz and the oscillations are damped with a time constant of (1/.3558) 2.81 Sec. for case a and the corresponding frequency and time constant are 1.73 Hz and 1.7 Sec for case b. When a six-phase machine has been considered to deliver power equal to that of three-phase machine, the corresponding values for case a and case b are obtained as 1.92 Hz, 1.2 Sec and 1.99 Hz, .936 Sec. respectively.

When the six-phase machine has been considered to deliver power twice that of three-phase machine, these values are 1.97 Hz, 1.707 Sec. and 2.35 Hz, 1.23 Sec. respectively.

From Table 2.4, when the machines without damper windings are considered, it is observed that the pair (λ_3, λ_4) and λ_5 are the eigenvalues which almost decide the dynamic stability of the system. For a three-phase machine, the pair (λ_3, λ_4) corresponds to the frequencies of approximately 1.47 Hz with time constant 6.06 Sec for case a, and 1.7 Hz, 7.4 Sec for case b. When the six-phase machine

delivers power equal to that of three-phase machine, the corresponding values are 1.89 Hz, 4.6 Sec. and 1.96 Hz and 4.6 Sec. for case a and case b respectively, while for the six-phase machine delivering power twice that of three-phase machine, corresponding values are 1.94 Hz, 3.61 Sec and 2.3 Hz, 4.16 Sec.

It is observed that λ_7 in Table 2.3 and λ_5 in Table 2.4 have shifted **left** from the origin more in case of six-phase synchronous machine while it delivers power equal to that of three-phase machine and even, when a six-phase machine delivers double the power, the corresponding eigen-values for the **different cases** of six-phase machine have better location than those of three-phase machine.

The initial value responses corresponding to $\Delta\delta \sim t$ and $\Delta\omega \sim t$ have been plotted and shown in Figs. 2.6–2.13 for the three-phase and six-phase machines delivering equal power for different cases with and without damper windings. It is observed from Figs. 2.6 and 2.8 that the oscillations in $\Delta\delta$ have damped out rapidly than those corresponding to three-phase machine for case a and case b. From Figs. 2.10 and 2.12, when the machines are considered without damper winding, the oscillations are very slowly dying out; however, in the case of six-phase machine, they die a little faster and with lower peak. From Figs. 2.7 and 2.9, the plot of $\Delta\omega \sim t$ for three-phase and six-phase

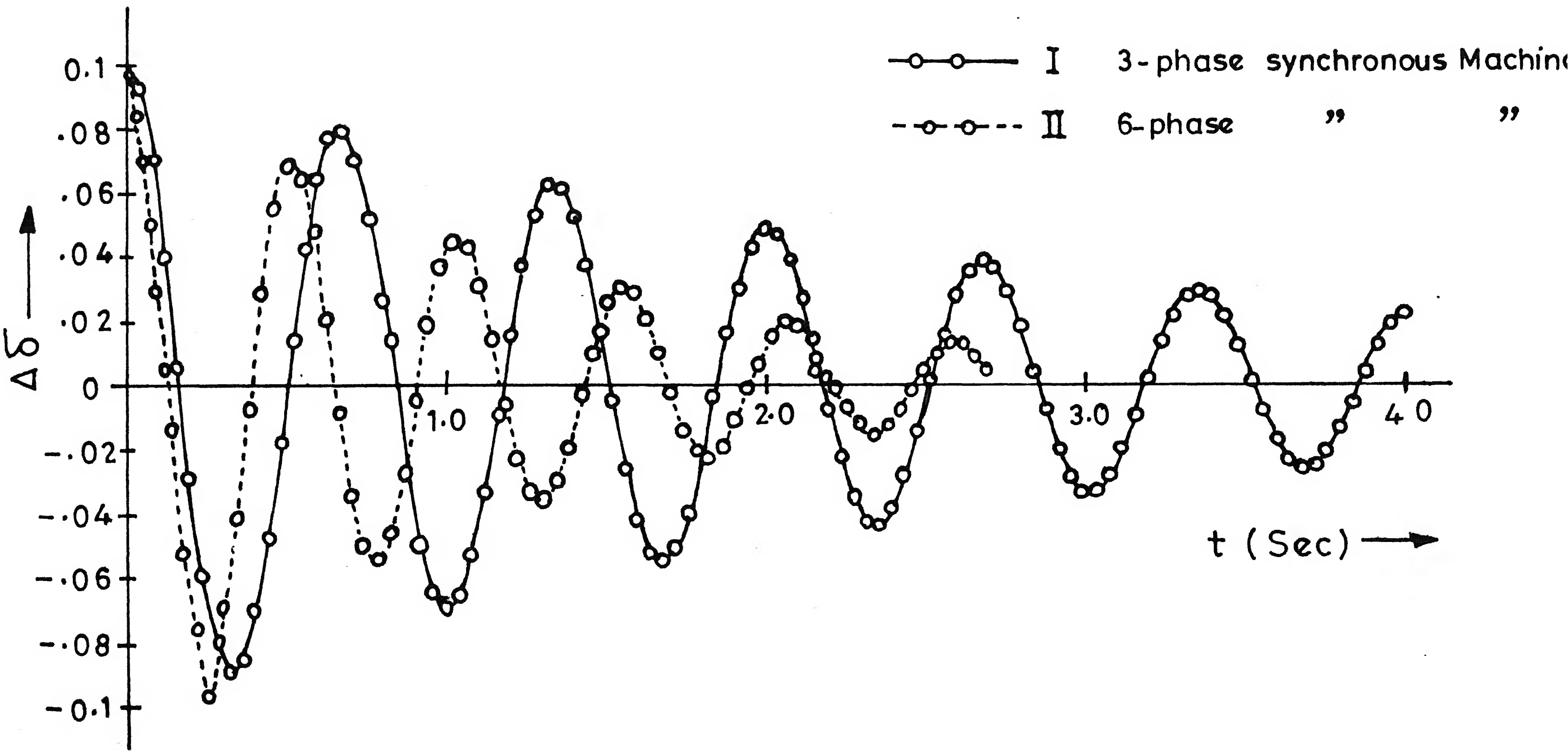


FIG. 2.6 INITIAL VALUE RESPONSE OF $\Delta\delta \sim t$ FOR 3-PHASE AND 6-PHASE MACHINES WITH DAMPER WINDINGS FOR Case a .

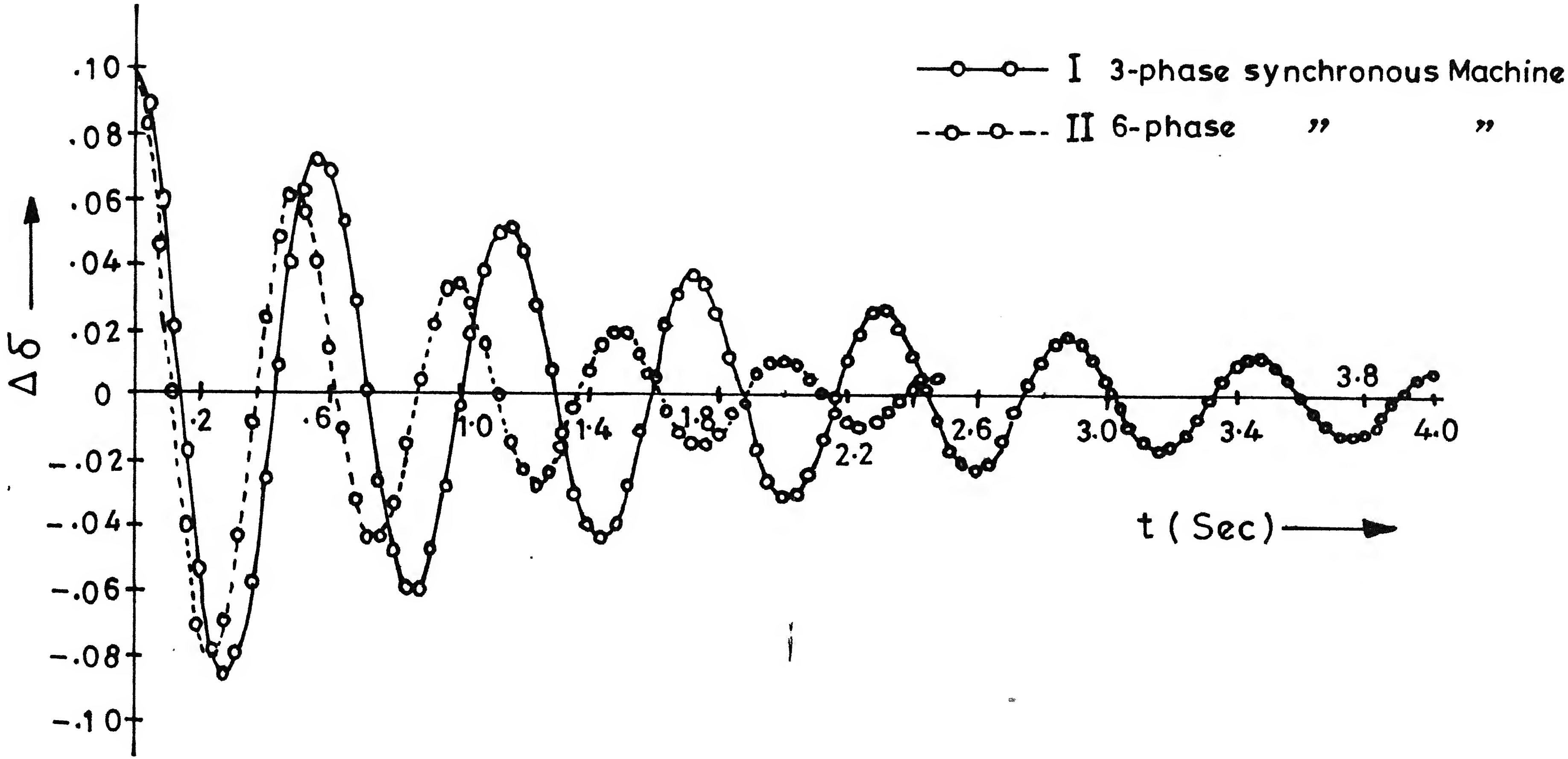


FIG.2.8 INITIAL VALUE RESPONSE OF $\Delta\delta \sim t$ FOR 3-PHASE AND 6-PHASE MACHINES WITH DAMPER WINDINGS FOR Case b.

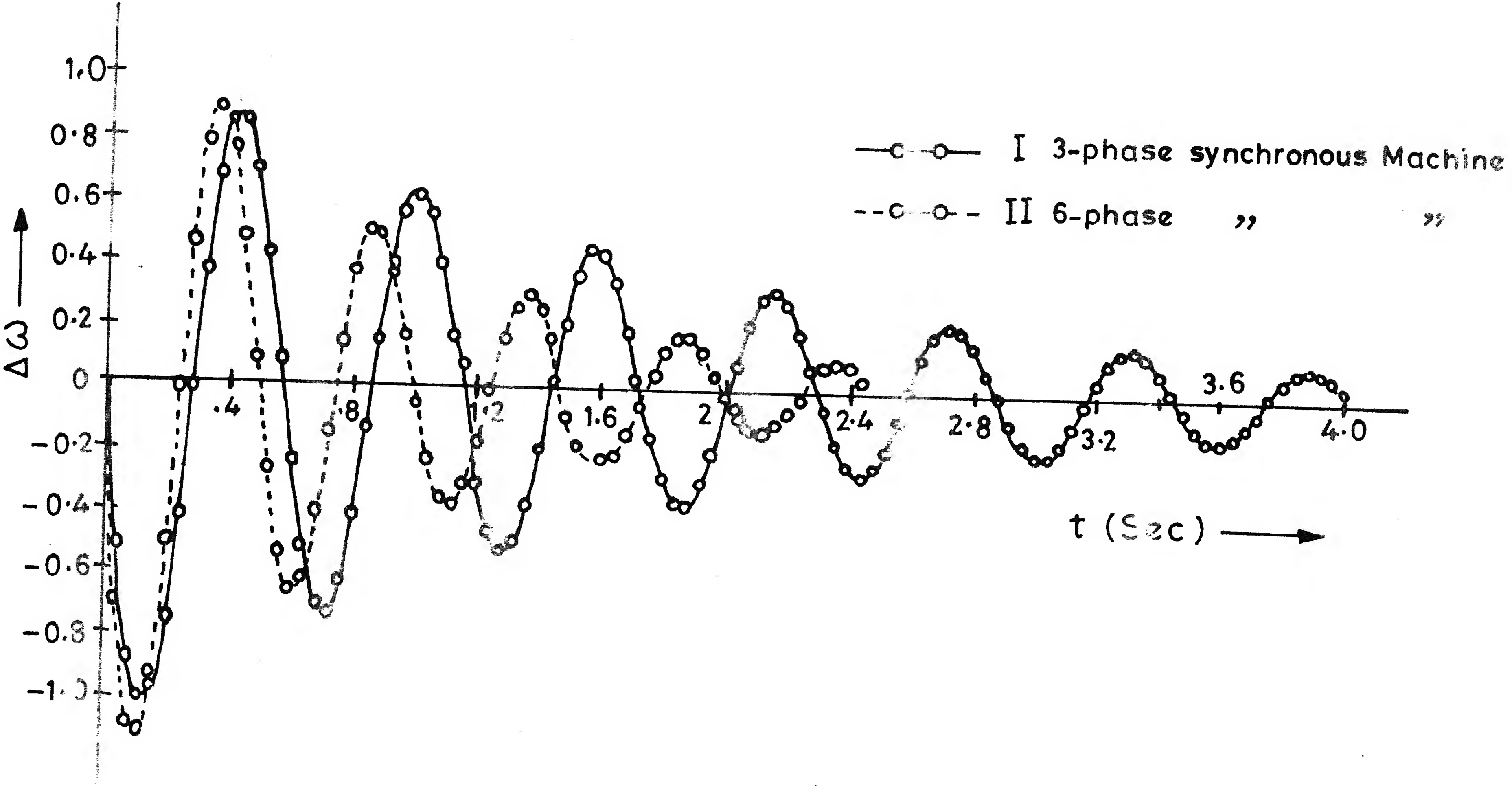


FIG. 2.9 INITIAL VALUE RESPONSE OF $\Delta\omega \sim t$ FOR 3-PHASE AND 6-PHASE MACHINES WITH DAMPER WINDING FOR Case b

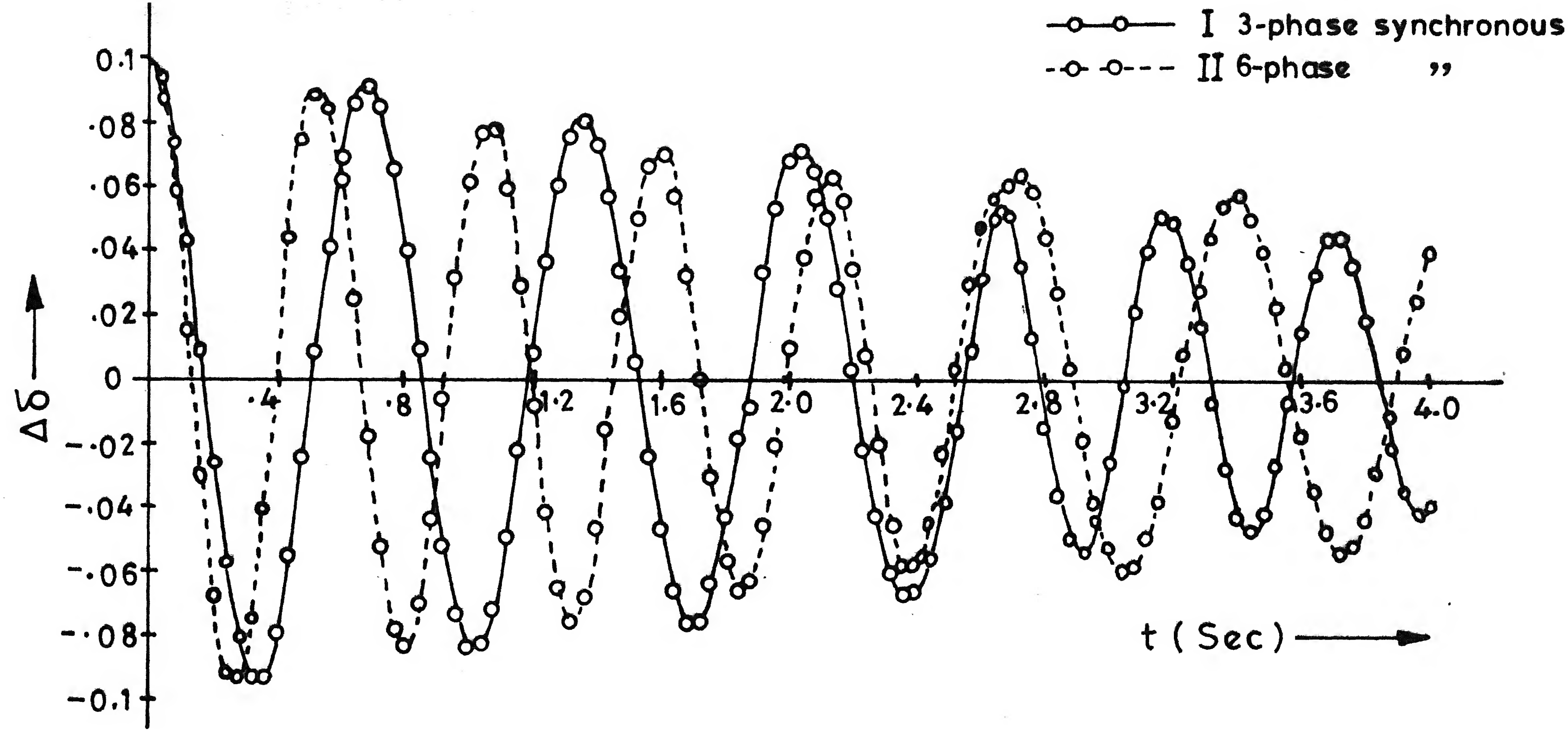


FIG.2.10 INITIAL VALUE RESPONSE OF $\Delta\delta$-t FOR 3-PHASE AND 6-PHASE MACHINES WITHOUT DAMPER WINDINGS FOR Case a.

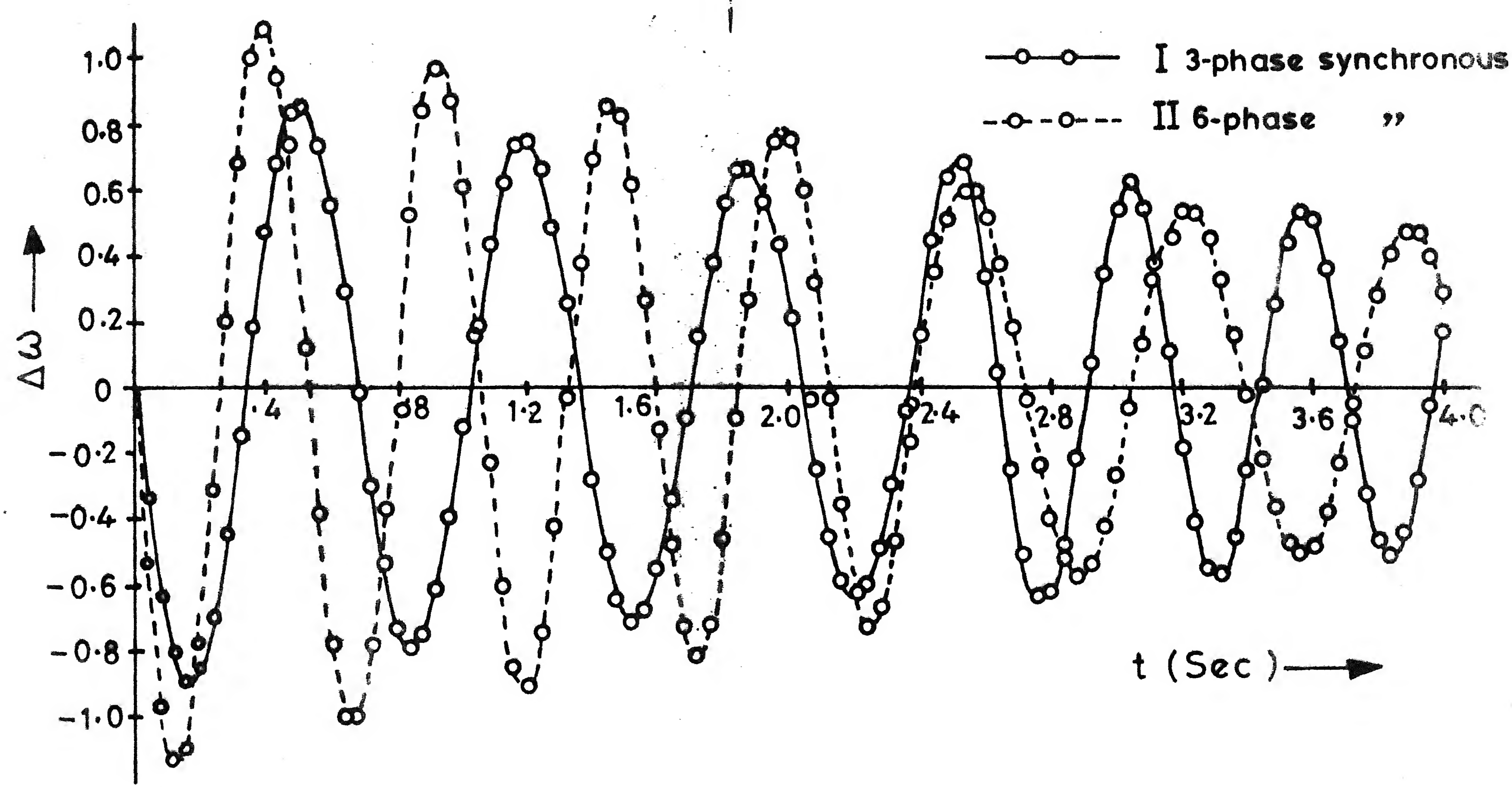
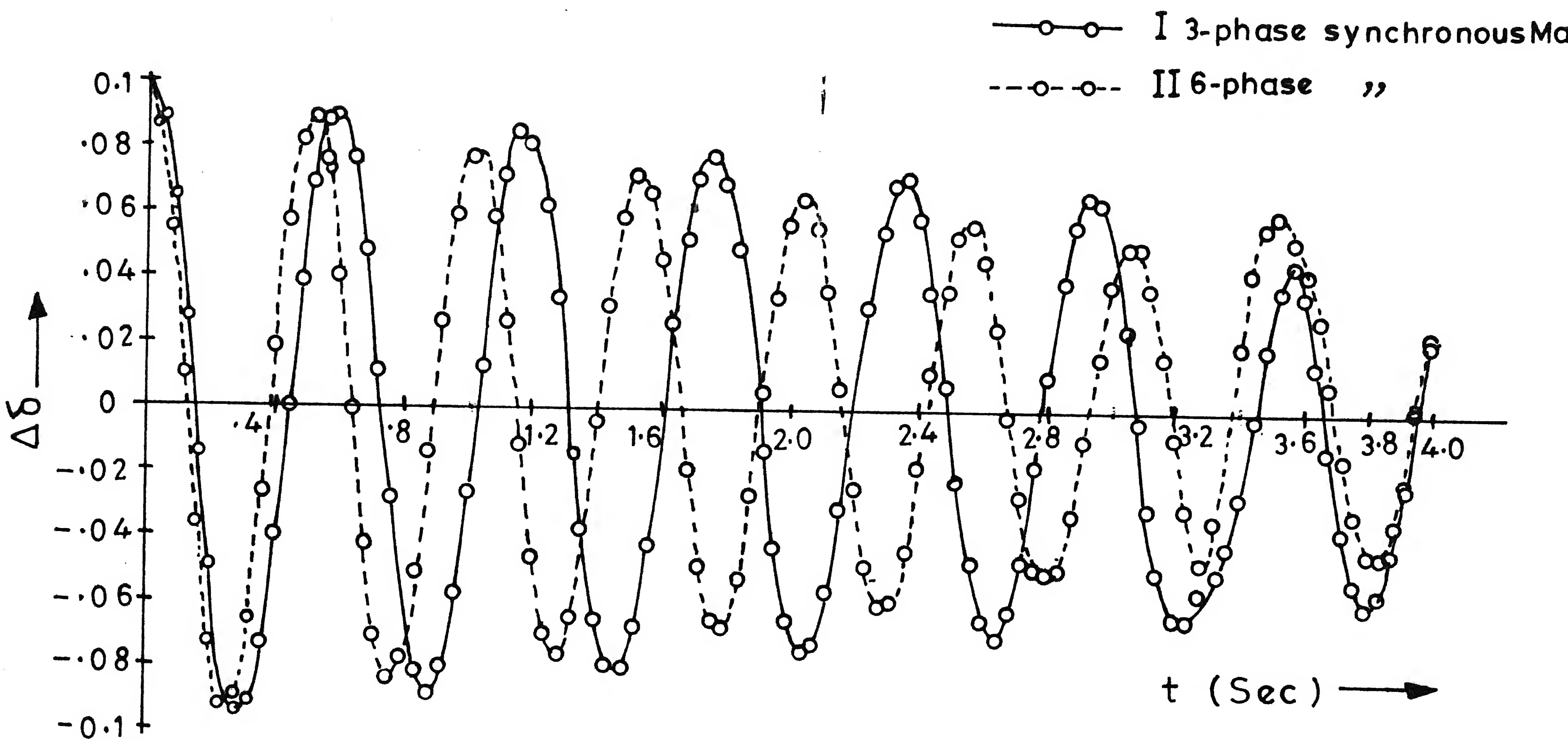


FIG. 2.11 INITIAL VALUE RESPONSE OF $\Delta\omega \sim t$ FOR 3-PHASE AND 6-PHASE MACHINES WITHOUT DAMPER WINDING FOR Case a.



FIG·2.12 INITIAL VALUE RESPONSE OF Δδ~t FOR 3-PHASE AND 6-PHASE MACHINES WITHOUT DAMPER WINDING FOR Case b.

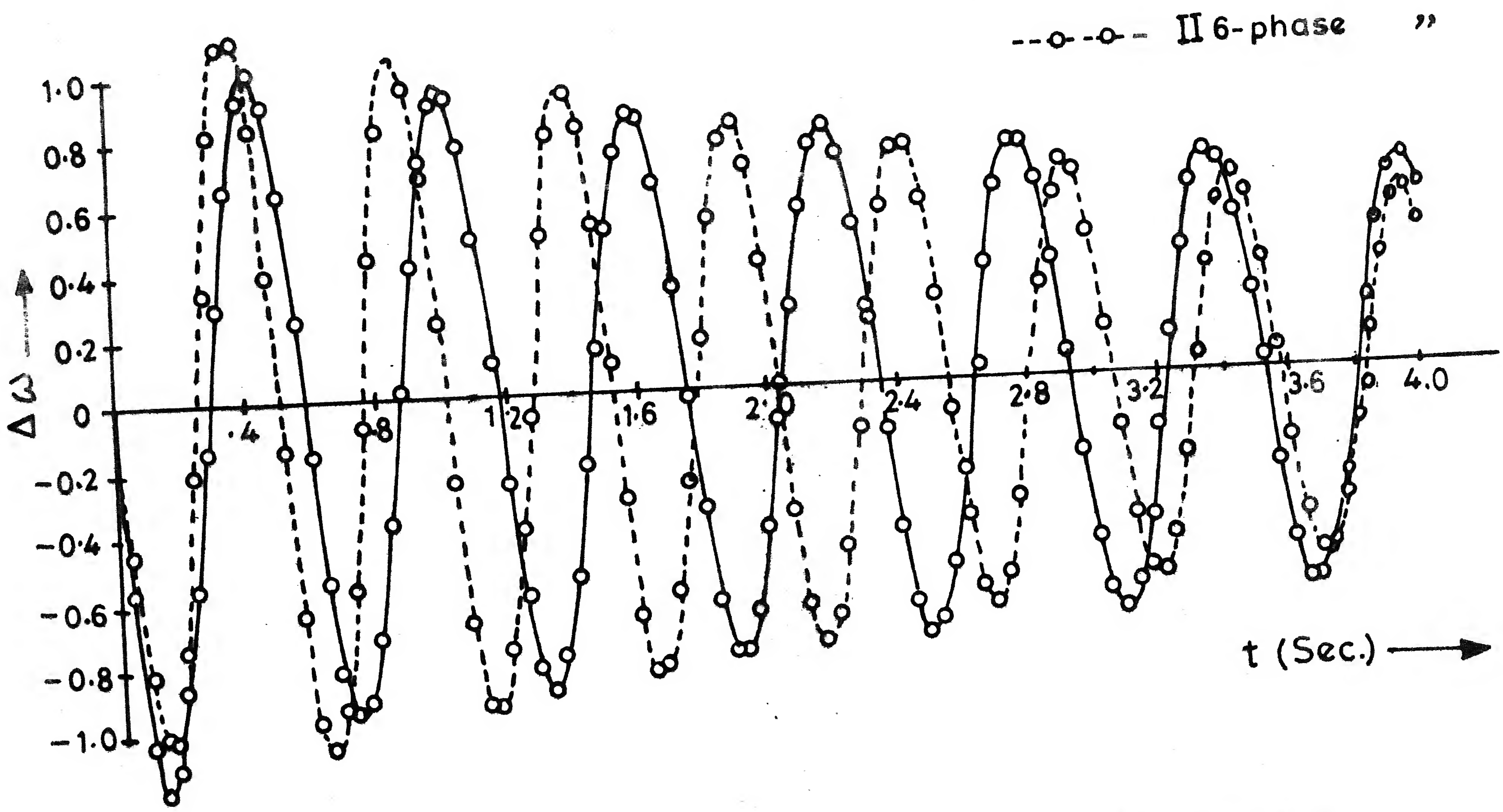


FIG.2.13 INITIAL VALUE RESPONSE OF $\Delta\omega \sim t$ FOR 3-PHASE AND 6-PHASE MACHINES WITHOUT DAMPER WINDING FOR Case b.

machines with damper windings for case a and case b, it is observed that oscillations die out faster in the case of six-phase machine, but with a higher peak for 1 cycle; and this is evident from the pair of eigenvalues (λ_3, λ_4) in Table 2.3. Further, it is observed from the Figs. 2.11 and 2.13, the oscillations tend to die out faster in case of six-phase machines but initially with higher peak for three-cycles.

2.5 CONCLUSION

The various inductances of a six-phase (multi-phase) synchronous machine have been obtained from the fundamentals. An orthogonal (linear and power invariant) transformation for a multi-phase system has been proposed to transform the phasor quantities into dqo component quantities in order to simplify the analysis of multi-phase synchronous machine. A linearized model has been developed for a six-phase machine to carry out the dynamic stability analysis of the six-phase synchronous machine. It is observed from the dynamic stability investigation that the six-phase machine while delivering power equal to that of a three-phase machine, is dynamically more stable than tne three-phase machine, and also when it delivers double power, the stability of six-phase machine has improved as compared to that of three-phase machine.

CHAPTER 3

GENERALISED MATHEMATICAL MODELLING OF n-phase POWER SYSTEM ELEMENTS

3.1 INTRODUCTION

In the previous chapter, a detailed dynamic model of a six-phase (multi-phase) synchronous machine has been developed and the dynamic stability of a six-phase (multi-phase) synchronous machine has also been investigated. In the current chapter, mathematical descriptions (models) of various n-phase (multi-phase) elements of power systems have been obtained, for the mathematical modelling is a very flexible and inexpensive tool for the in-depth and the detailed analysis of the behaviour of systems. Usually the analysis of three-phase system is conducted on the single-phase basis assuming balanced conditions; the power system is normally not balanced due to the various causes, viz., long untransposed transmission line, bundled conductors, single-phase switching, unbalanced loading, etc. Therefore, in order to take these factors into consideration, it is necessary to develop the mathematical model in the phase frame of reference.

A mathematical description of a n-phase (multi-phase) synchronous machine has been developed in section 3.2 incorporating imbalances of the machine. If the power is transmitted through a n-phase transmission line, it will require a three-phase/n-phase transformer at each end of the

line requiring thereby a mathematical model for such transformers.

In section 3.3, three-phase/n-phase transformer models have been developed for different types of winding connections. This is done by first obtaining the voltage and current relations of the two sides of the windings of three-phase/n-phase transformers in section 3.3.1, and using these relations, the nodal representations of the transformers have been obtained in section 3.3.2.

Further, mathematical models for multi-phase transmission line have been given in section 3.4 in terms of phase impedance matrix, π-circuit representation and ABCD parameters. If the three-phase transmission line is unbalanced and the interest of investigation lies on three-phase side of the line, it is expedient to replace the balanced n-phase line associated with the transformers by their equivalent three-phase parameters to carryout the analysis. The equivalent three-phase representation of such n-phase (multi-phase) line has been derived in section 3.5. The equivalent three-phase impedance has been derived in section 3.5.1 and the equivalent three-phase ABCD parameter representing π-circuit in section 3.5.2. Further, for the composite system to be completely balanced, only single-phase parameters are required. The equivalent single-phase π circuit parameters have been derived

from the equivalent three-phase ABCD parameters in section 3.6.

When a three-phase transmission line is connected between the two n-phase (multi-phase) transmission lines through n-phase/three-phase transformers, it is necessary to obtain the equivalent n-phase representation of the three-phase transmission line associated with two transformers at both the ends. These parameters are desirable when the analysis of the composite system is carried out on n-phase basis. The equivalent n-phase parameters in impedance form have been derived in section 3.7.1 and those in ABCD parameters representing π circuit in section 3.7.2.

Finally, the chapter concludes in section 3.8.

3.2 MULTI-PHASE SYNCHRONOUS MACHINE MODEL

This section presents a mathematical description of a multi-phase synchronous machine in phasor co-ordinates for steady-state analysis under balanced as well as under unbalanced conditions. In order to develop the model, a schematic representation of a star connected multi-phase (n-phase) synchronous machine, as shown in Fig. 3.1, has been considered. The generated (internal) voltages of the machine are assumed to be balanced, and the terminal voltages unbalanced as the network connected to its terminal (i.e. load) are unbalanced.

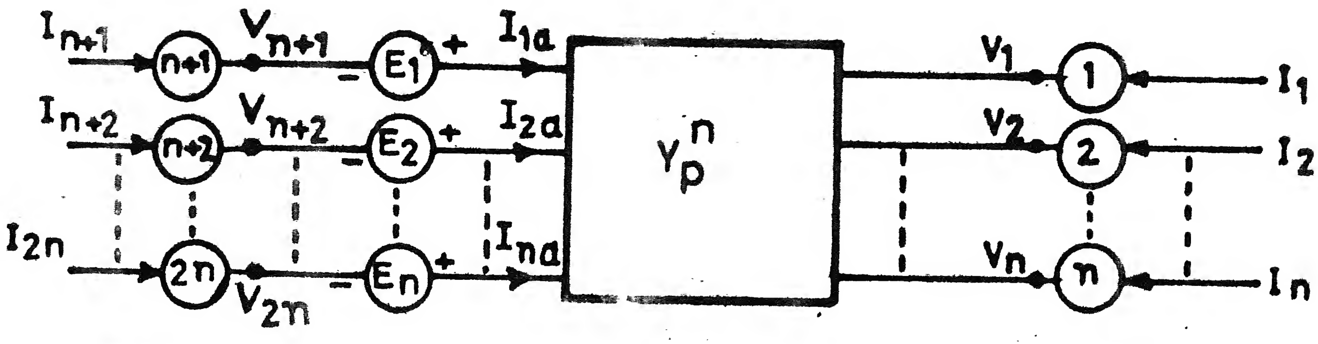


FIG.3.1 Schematic reprentation of a n-phase synchronous machine.

Let $E_1, E_2, \ldots, E_n$ be a balanced set of internal voltages of a n-phase synchronous machine, and nodes n+1, n+2,...,2n be joined together to form a star (i.e., neutral) point; a n-phase synchronous machine can thus be represented by a balanced set of internal voltages behind synchronous phase impedances.

Then, from Fig. 3.1, the voltage equations in the impedance form are,

$$V^n_{P'} + E^n_P = Z^n_P \, I^n_{P'} + V^n_P \qquad (3.1)$$

where

$$V^n_{P'} = [\ V_{n+1} \quad V_{n+2} \quad \cdots \quad V_{2n}\ ]^t$$

$$V^n_P = [\ V_1 \ V_2 \quad \cdots \quad V_n\ ]^t$$

$$E^n_P = [\ E_1 \ E_2 \quad \cdots \quad E_n\ ]^t$$

$$I^n_{P'} = [\ I_{1a} \ I_{2a} \quad \cdots \quad I_{na}\ ]^t$$

and

Z^n_P = (nxn) impedance i.e. coefficient matrix of the n-phase synchronous machine $(= [\ Y^n_P\ ]^{-1})$

For the voltage and current directions as shown in Fig. 3.1, the nodal representation of the machine can be written as,

$$\begin{bmatrix} I_p^n \\ \\ I_{p'}^n \end{bmatrix} = \begin{bmatrix} Y_p^n & -Y_p^n & -Y_p^n \\ \\ -Y_p^n & Y_p^n & Y_p^n \end{bmatrix} \begin{bmatrix} V_p^n \\ V_{p'}^n \\ E_p^n \end{bmatrix} \qquad (3.2)$$

where

$$I_p^n = [\, I_1 \quad I_2 \quad \cdots \quad I_n]^t$$

and

$Y_p^n = [\, Z_p^n]^{-1}$ = Admittance matrix of the n-phase synchronous machine of order (nxn).

Now, for a balanced set of internal voltages,

$$E_p^n = [\, 1 \quad \alpha^* \quad \alpha^{2*} \quad \cdots \quad \alpha^{(n-1)*}]^t \; E_1 \qquad (3.3)$$

where

$$\alpha = e^{j2\pi/n} = \cos(2\pi/n) + j\sin(2\pi/n) = 1\underline{/2\pi/n} = n\sqrt{1}$$

The voltage vector E_p^n in eqn. (3.3) can also be written as

$$E_p^n = \begin{cases} [\, 1 \quad \alpha^* \quad \alpha^{2*} \quad \cdots \quad \alpha^{(\frac{n}{2}-1)*} \quad -1 \quad -\alpha^* \quad -\alpha^{2*}, \cdots \quad \alpha^{(\frac{n}{2}-1)*}]^t \; E_1 & (3.4a) \\ \quad \text{when n is an even number} \\ \text{or} \\ [1 \quad \alpha^* \quad \alpha^{2*} \quad \cdots \quad \alpha^{(\frac{n+1}{2}-1)*} \quad \alpha^{(\frac{n+1}{2}-1)} \quad \alpha^{(\frac{n+1}{2}-2)} \quad \cdots \quad \alpha]^t \; E_1 & (3.4b) \\ \quad \text{when n is an odd number} \end{cases}$$

If the nodes n+1 to 2n be joined together to form the star point, then we have,

$$I_N = I_{n+1} + I_{n+2} + \cdots + I_{2n} \tag{3.5}$$

and

$$V_N = V_{n+1} = V_{n+2} = \cdots = V_{2n} \tag{3.6}$$

The internal voltages can then be given by

$$E_p^n = [E_1 + V_N \quad E_2+V_N \ \ldots \ E_n+V_N]^t \tag{3.7}$$

and the currents I_p^n, by

$$I_{1a} = \frac{S_1^*}{(E_1+V_N)^*} ;\ I_{2a} = \frac{S_2^*}{(E_2+V_N)^*} ;\ \ldots ;\ I_{na} = \frac{S_n^*}{(E_n+V_N)^*} ; \tag{3.8}$$

where $S_1, S_2, \ldots, S_n$ are complex powers for n phases (1 to n) respectively, as these are unequal.

Further the (nxn) admittance matrix Y_p^n in equation (3.2) is given by

$$Y_p^n = \begin{bmatrix} Y_s & Y_{m1} & Y_{m2} & \cdot & Y_{m,i-1} & \cdot\cdot & Y_{m,n-1} \\ Y_{m,n-1} & Y_s & Y_{m1} & \cdot & Y_{m,i-2} & \cdot\cdot & Y_{m,n-2} \\ Y_{m,n-2} & Y_{m,n-1} & Y_s & \cdot & Y_{m,i-1} & \cdot\cdot & Y_{m,n-3} \\ \cdot & \cdot & \cdot & \cdot & & \cdot\cdot & \cdot \\ Y_{m,n-i+1} & Y_{m,n-i+2} & Y_{m,n-i+3} & \cdot & Y_s & \cdot\cdot & Y_{m,n-i} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ Y_{m1} & Y_{m2} & Y_{m3} & \cdot & Y_{mi} & \cdot\cdot & Y_s \end{bmatrix} \tag{3.9}$$

If the data are available in symmetrical component form, its value in the phasor form can be obtained by the relation

$$[Y_P^n] = [T_S^n][Y^n_{comp}][T_S^n]^* \tag{3.10}$$

where

$$[Y^n_{comp}] = \text{diag}\,[\,y_o \quad y_1 \quad y_2 \quad \cdots \quad y_{n-1}\,]\;;$$

$y_o, y_1, \ldots, y_{n-1}$ are n-phase sequence admittances,

and

$[T_S^n]$ is the generalized symmetrical component transformation matrix for n-phase system which can however be developed by exploiting the cyclic symmetries inherent in the network using the group theoretic technique, and it can be written as follows.

$$T_S^n = \frac{1}{\sqrt{n}}\begin{bmatrix}
1 & 1 & 1 & 1 & \cdots & 1 & \cdots & 1 & 1 & 1 \\
1 & \alpha^{*} & \alpha^{2*} & \alpha^{3*} & \cdots & \alpha^{(j-1)*} & \cdots & \alpha^{(\ -3)*} & \alpha^{(n-2)*} & \alpha^{(n-1)*} \\
1 & \alpha^{2*} & \alpha^{4*} & \alpha^{6*} & \cdots & \alpha^{2(j-1)*} & \cdots & \alpha^{(n-6)*} & \alpha^{(n-4)*} & \alpha^{(n-2)*} \\
1 & \alpha^{3*} & \alpha^{6*} & \alpha^{9*} & \cdots & \alpha^{3(j-1)*} & \cdots & \alpha^{(n-9)*} & \alpha^{(n-6)*} & \alpha^{(n-3)*} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots \\
1 & \alpha^{(i-1)*} & \alpha^{2(i-1)*} & \alpha^{3(i-1)*} & \cdots & \alpha^{(i-1)(j-1)*} & \cdots & \alpha^{(-3i+3)*} & \alpha^{(-2i+2)*} & \alpha^{(-i+1)*} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots \\
1 & \alpha^{(n-3)*} & \alpha^{(n-6)*} & \alpha^{(n-9)*} & \cdots & \alpha^{(-3j+3)*} & \cdots & \alpha^{9*} & \alpha^{6*} & \alpha^{3*} \\
1 & \alpha^{(n-2)*} & \alpha^{(n-4)*} & \alpha^{(n-6)*} & \cdots & \alpha^{(-2j+2)*} & \cdots & \alpha^{6*} & \alpha^{4*} & \alpha^{2*} \\
1 & \alpha^{(n-1)*} & \alpha^{(n-2)*} & \alpha^{(n-3)*} & \cdots & \alpha^{(-j+1)*} & \cdots & \alpha^{3*} & \alpha^{2*} & \alpha^{*}
\end{bmatrix} \tag{3.11}$$

where the operator $\alpha = \sqrt[n]{1} = 1\underline{/2\pi/n} = e^{j2\pi/n}$

Using equations (3.10) through (3.11), the various elements of the coefficient matrix in equation (3.9) can be obtained which are as follows.

$$y_s = \frac{1}{n} [y_o + \sum_{i=1}^{n-1} y_i] \qquad (3.12a)$$

$$y_{m1} = \frac{1}{n} [y_o + \sum_{i=1}^{n-1} \alpha^i y_i] \qquad (3.12b)$$

$$y_{m2} = \frac{1}{n} [y_o + \sum_{i=1}^{n-1} \alpha^{2i} y_i] \qquad (3.12c)$$

$$y_{m3} = \frac{1}{n} [y_o + \sum_{i=1}^{n-1} \alpha^{3i} y_i] \qquad (3.12d)$$

$$\vdots \qquad \vdots \qquad\qquad \vdots$$

$$y_{m,j-1} = \frac{1}{n} [y_o + \sum_{i=1}^{n-1} \alpha^{(j-1)i} y_i] \qquad (3.12i)$$

$$\vdots \qquad \vdots \qquad\qquad \vdots$$

$$y_{m,n-2} = \frac{1}{n} [y_o + \sum_{i=1}^{n-1} \alpha^{(n-2)i} y_i] \qquad (3.12m)$$

$$y_{m,n-1} = \frac{1}{n} [y_o + \sum_{i=1}^{n-1} \alpha^{(n-1)i} y_i] \qquad (3.12n)$$

Substituting the values of E_p^n, I_p^n, and Y_p^n from equations (3.7)-(3.9) respectively in equation (3.2), the following representation of the n-phase synchronous machine in matrix form is obtained.

$$\begin{bmatrix} & -y_o & -y_1 \\ & -y_o & -\alpha^{*} y_1 \\ Y_P^n & \cdot & \cdot \\ & \cdot\cdot & \cdot \\ & \cdot & \cdot \\ & -y_o & \alpha^{(n-1)*} y_1 \\ & y_o & y_1 \\ & y_o & \alpha^{*} y_1 \\ Y_P^n & \cdot & \cdot \\ & \cdot & \cdot \\ & y_o & \alpha^{(n-1)*} y_1 \end{bmatrix} \begin{bmatrix} V_P^n \\ \\ \\ V_N \\ \\ E_1 \end{bmatrix} = \begin{bmatrix} I_P^n \\ \frac{S_1^*}{(E_1+V_N)^*} \\ \frac{S_2^*}{(E_2+V_N)^*} \\ \vdots \\ \frac{S_n^*}{(E_n+V_N)^*} \end{bmatrix} \tag{3.13}$$

By adding the **last** n equations of (3.13), an additional equation for node N (neutral node i.e. star point) is obtained as

$$I_N = [\,-y_o \quad -y_o \quad \ldots \quad ny_o \qquad o\,]\,[\,V_P^n \quad V_N \quad E_1\,]^t \tag{3.14}$$

From equation (3.3),

$$E_P^{n*} = [\,1 \quad \alpha \quad \alpha^2 \quad \ldots \quad \alpha^{(n-1)}\,]^t \; E_1^* \tag{3.15}$$

cross multiplication of the last n equations of (3.13), and their addition and substitution of different element of admittance matrix in equation (3.2) gives the following expression after simplification.

$$-y_o V_N^* \sum_{i=1}^{n} V_i + (ny_o + y_N) V_N^* V_N - y_1 E_1^* \sum_{i=1}^{n} \alpha^{i-1} V_i$$

$$+ ny_1 E_1^* = \sum_{i=1}^{n} S_i^* \qquad (3.16)$$

In equation (3.16), (n+1) terms are V_N^* times I_N and the injected neutral current $I_N = 0$. Taking this point into account, the equation (3.16) can further be simplified as

$$[\, -1 \;\; -\alpha \;\; -\alpha^2 \;\ldots\; -\alpha^{n-3} \;\; -\alpha^{n-2} \;\; -\alpha^{n-1} \;\; n\,]\; y_1 \begin{bmatrix} V_P^n \\ \\ E_1 \end{bmatrix}$$

$$= \frac{\sum_{i=1}^{n} S_i^* + y_N |V_N|^2}{E_1^*} \qquad (3.17)$$

combining the first n equations of (3.13) to (3.17) and taking $I_N = 0$, the machine equations in the matrix form become,

$$\begin{bmatrix} & & -y_1 & -y_o \\ Y_P^n & & -\alpha^* y_1 & -y_o \\ & & -\alpha^{2*} y_1 & -y_o \\ & & \cdot & \cdot \\ & & \cdot & \cdot \\ -y_1 \;\; -\alpha y_1 \;\cdot\cdot -\alpha^{n-1} y_1 & & ny_1 & 0 \\ -y_o \;\; -y_o \;\cdot\cdot -y_o & & 0 & ny_o + y_N \end{bmatrix} \begin{bmatrix} V_P^n \\ \\ \\ \\ \\ E_1 \\ V_N \end{bmatrix} = \begin{bmatrix} I_P^n \\ \\ \\ \\ \\ \left(\sum_{i=1}^{n} S_i^* + y_N |V_N|^2\right) / E_1^* \\ 0 \end{bmatrix}$$

(3.18)

As the term $Y_N|V_N|^2$ is very small in comparision to $\sum_{i=1}^{n} S_i^*$, the equation (3.18) reduces to the following equation after neglecting this term.

$$\begin{bmatrix} Y_P^n & -\bar{Y}_1 & -\bar{Y}_o \\ -\bar{Y}_2 & ny_1 & 0 \\ -\bar{Y}_o^t & 0 & (ny_o+y_N) \end{bmatrix} \begin{bmatrix} V_P^n \\ E_1 \\ V_N \end{bmatrix} = \begin{bmatrix} I_P^n \\ \sum_{i=1}^{n} S_i^*/E_1^* \\ 0 \end{bmatrix} \qquad (3.19)$$

where,

$$\bar{Y}_1 = [1 \quad \alpha^* \quad \alpha^{2*}, \ldots \alpha^{(n-3)*} \quad \alpha^{(n-2)*} \quad \alpha^{(n-1)*}]^t \, y_1$$

$$\bar{Y}_2 = [1 \quad \alpha \quad \alpha^2, \ldots \alpha^{n-3} \quad \alpha^{n-2} \quad \alpha^{n-1}] \, y_1$$

and

$$\bar{Y}_o = [1 \quad 1 \quad 1 \quad 1 \quad .. \quad 1 \quad 1]^t \, y_o$$

Thus equation (3.19) represents a generalized mathematical model for multi-phase synchronous machine. It calculates the terminal phase voltages, either balanced or unbalanced of synchronous machine. It makes use of the total power of the machine and not the individual power of each phase. Therefore, in order to determine the **left** hand vector in equation (3.19), individual phase power will not be required. In the right hand side, the current vector I_P^n is the injected current, so, if there is no local load on the synchronous machine bus, the

injected currents are zero. If, however, there are local loads on the machine, injected current cannot be taken as zero, rather one nth of the total local load can be assigned to each of the phases. Due to the imbalances in the terminal voltages because of unbalanced terminal currents, each phase power is not equal to one nth of the total power thereby precluding the calculation of phase currents.

3.3 THREE-PHASE/MULTI-PHASE (n-PHASE) TRANSFORMER MODEL

The power system networks will essentially require three-phase/n-phase transformers for multi-phase transmission line in order to be compatible with the three-phase synchronous generator and the three-phase loads. Therefore, mathematical models for such transformers will be needed to analyse the over all system. Therefore, this section is devoted to the nodal representation of three-phase/n-phase transformers by taking Wye or delta windings for three-phase side and star connection for n-phase side of the transformer by presenting the voltage and current relations of the two sides windings of a transformer with the tappings on both the sides, and therefrom various particular cases can be derived.

3.3.1 Voltages and Current Relations of Three-Phase/n-Phase Transformers

The voltage and the current relations for primary and secondary sides of the three-phase and six-phase transfor-

mers are available in the literature [8,22,23]. Although a three-phase/six-phase transformer is simple in construction, yet it is not so simple to construct a three-phase/twelve-phase transformers. If, however, it is realized, it is required to be especially built. Moreover, the constructional complexity will be more involved for a three-phase/n-phase transformer, where n is larger than twelve. In order to develop the voltage and current relations for the two sides of three-phase/n-phase transformers, an equivalent n-phase transformation has been obtained from a given three-phase supply based upon the following principle.

'The desired n-phase system can be obtained with the help of an appropriate number of three-phase/six-phase transformers. The primary voltages are obtained for the different transformers from the given three-phase supply by getting it (three-phase supply) shifted through proper angles by phase shifting circuits'.

The number of required transformers will be n/6 and the angle of shift for the different primaries will be $0, m, \ldots, (\frac{\pi}{3} - m)$ where $m = 2\pi/n$.

Based upon this principle, three-phase (Wye)/n-phase (star) transformation has been obtained as shown in Fig. 3.2. For this case, the induced voltages at the secondary side can be represented as

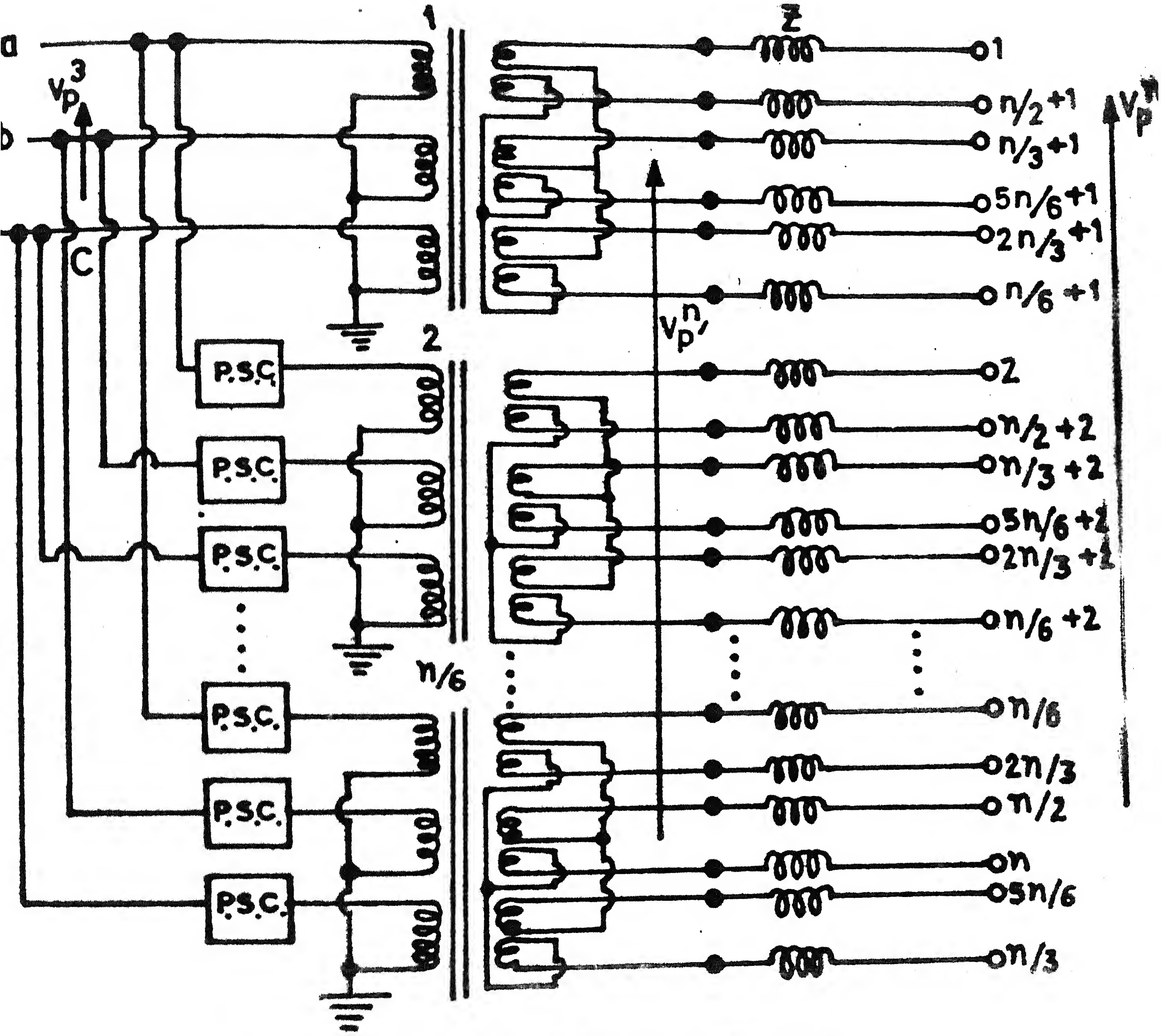


FIG. 3.2 Schematic diagram for 3-phase/n-phase transformer

$$
\begin{bmatrix} V'_1 \\ V'_2 \\ \vdots \\ V'_{n/6} \\ V'_{\frac{n}{6}+1} \\ V'_{\frac{n}{6}+2} \\ \vdots \\ V'_{\frac{n}{3}} \\ V'_{\frac{n}{3}+1} \\ V'_{\frac{n}{3}+2} \\ \vdots \\ V'_{\frac{n}{2}} \\ V'_{\frac{n}{2}+1} \\ V'_{\frac{n}{2}+2} \\ \vdots \\ V'_{\frac{2n}{3}} \end{bmatrix}
=
\begin{bmatrix} 1 & 0 & 0 \\ 1\underline{/-m} & 0 & 0 \\ \vdots & \vdots & \vdots \\ 1\underline{/-(\pi/3-m)} & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1\underline{/-m} \\ \vdots & \vdots & \vdots \\ 0 & 0 & -1\underline{/-(\pi/3-m)} \\ 0 & 1 & 0 \\ 0 & 1\underline{/-m} & 0 \\ \vdots & \vdots & \vdots \\ 0 & 1\underline{/-(\pi/3-m)} & 0 \\ -1 & 0 & 0 \\ -1\underline{/-m} & 0 & 0 \\ \vdots & \vdots & \vdots \\ -1\underline{/-(\pi/3-m)} & 0 & 0 \end{bmatrix}
\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad (3.20)
$$

$$
\begin{bmatrix} V'_{\frac{2n}{3}+1} \\ V'_{\frac{2n}{3}+2} \\ \vdots \\ V'_{5n/6} \\ V'_{5n/6+1} \\ \vdots \\ V'_{n} \end{bmatrix}
\quad
\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1\underline{/-m} \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1\underline{/-(\pi/3-m)} \\ 0 & -1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & -1\underline{/-(\pi/3-m)} & 0 \end{bmatrix}
$$

Equation (3.20) can be written in the compact form as

$$V^n_{P'} = NV^3_P \tag{3.21}$$

where N is the coefficient matrix of $[\,V_a \quad V_b \quad V_c\,]^t$ in equation (3.20).

The different voltages of the secondary side and the given supply voltage are shown in Fig. 3.3.

Thus, the voltage and the current relations at the terminals of an ideal transformer are

$$V^n_{P'} = [\,N\,]\,V^3_P \quad ; \quad V^3_P = [\,N\,]^{-1}\;V^n_{P'} \tag{3.22}$$

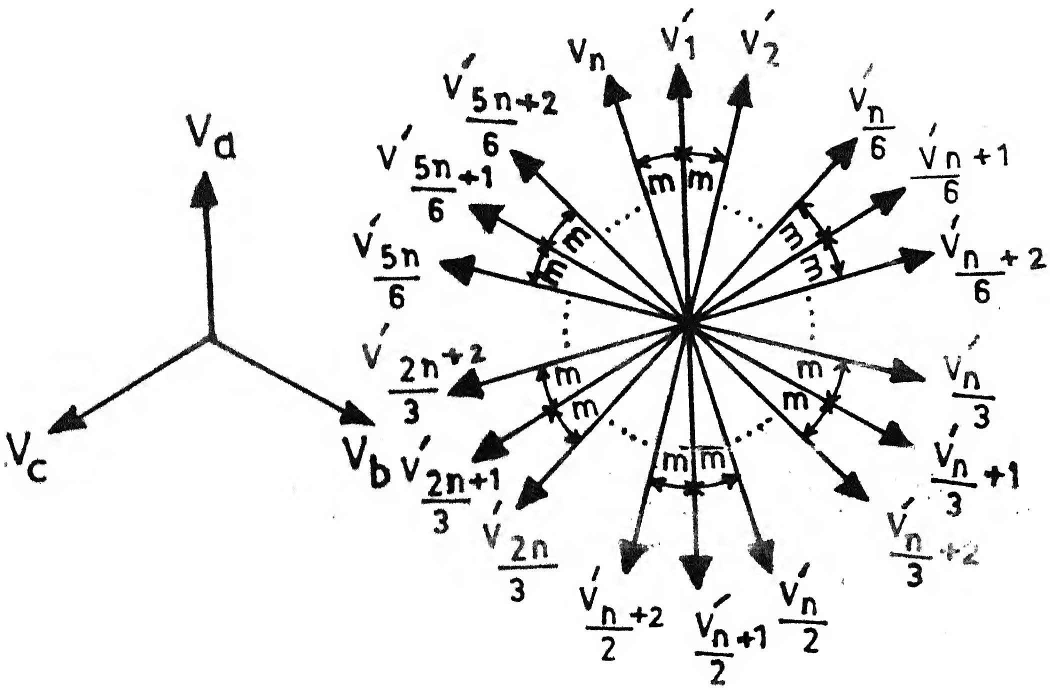


FIG. 3.3 Voltage of the given supply and the secondary sides at the different nodes

and

$$I_P^n = [N] I_P^3 \quad ; \quad I_P^3 = [N]^{-1} \; I_P^n \tag{3.23}$$

Since [N] is a rectangular matrix of order (nx3), its inverse cannot be determined. Therefore, by taking its Pseudo-inverse [60], equations (3.22) and (3.23) become

$$V_{P'}^n = [N] V_P^3 \quad ; \quad V_P^3 = \frac{1}{K_m} [N]^t \; V_P^n \tag{3.24}$$

and

$$I_P^n = [N] I_P^3 \quad ; \quad I_P^3 = \frac{1}{K_m} [N]^t \; I_P^n \tag{3.25}$$

where,

$$K_m = 2 \sum_{i=1}^{m_1} \underline{/-2\theta_i} \quad ; \quad m_1 = \frac{n}{6}$$

and θ_i is the angle difference between the primary of ith three-phase/six-phase transformer and the given three-phase supply.

For a particular case of three-phase Wye/six-phase star connected transformer, we have

$$[N]^t = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{bmatrix} \text{ and } K_m = 2 \tag{3.26}$$

If the off-nominal turn ratio α_{3t} in the primary side of the transformer is also taken into account as shown in the fig. 3.4, the equations (3.24) and (3.25) become

$$V^n_{P'} = \frac{1}{\alpha_{3t}} [N]\, V^3_P \;;\quad V^3_P = \frac{\alpha_{3t}}{K_m} [N]^t\, V^n_{P'} \tag{3.27}$$

and

$$I^n_P = \alpha_{3t} [N]\, I^3_P \;;\quad I^3_P = \frac{1}{\alpha_{3t} K_m} [N]^t\, I^n_P \tag{3.28}$$

If, however, the tappings are provided on both sides of the transformer i.e. three-phase and n-phase sides, the voltage and current relations corresponding to equations (3.27) and (3.28) will be

$$\frac{V^n_{P'}}{\alpha_{nt}} = \frac{1}{\alpha_{3t}} [N]\, V^3_P \;;\quad V^3_P = \frac{\alpha_{3t}}{\alpha_{nt}} \frac{1}{K_m} \cdot [N]^t\, V^n_{P'} \tag{3.29}$$

$$\alpha_{nt}\, I^n_P = \alpha_{3t} [N]\, I^3_P ;\quad I^3_P = \frac{\alpha_{nt}}{\alpha_{3t}} \frac{1}{K_m} \cdot [N]^t\, I^n_P \tag{3.30}$$

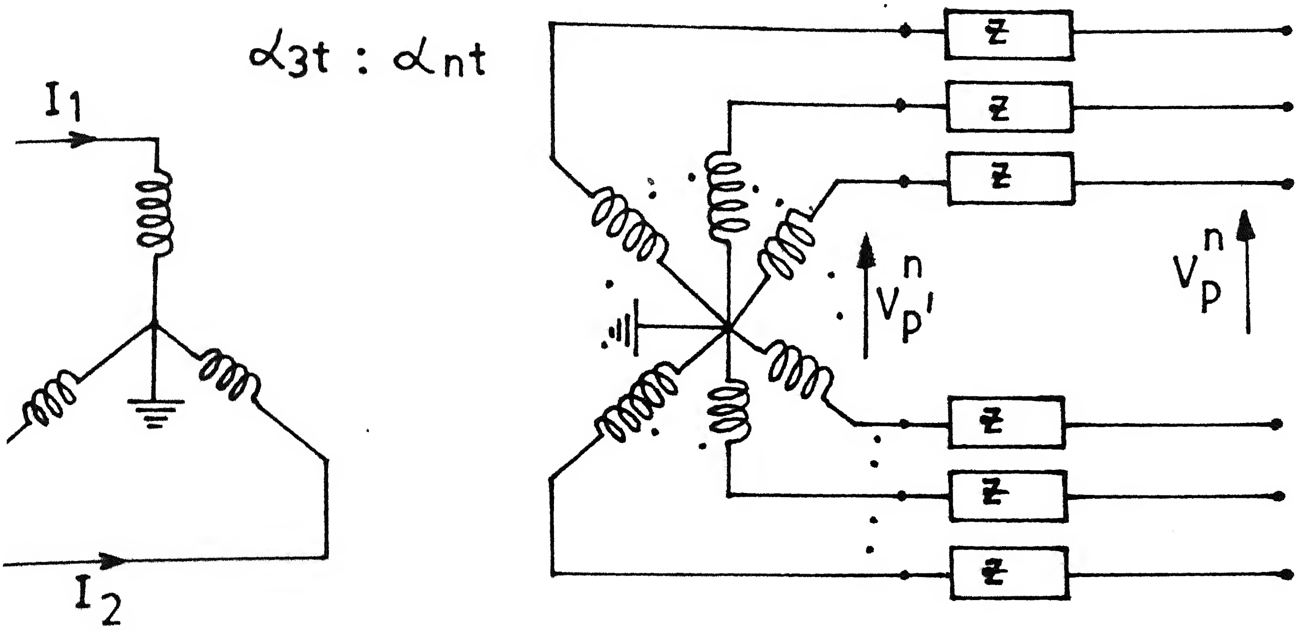
Further, a three-phase delta/n-phase star connected transformer of turn ratio α_{3t}:1 as shown in Fig. 3.5 is considered. Assuming nominal turn ratio, the following voltage and current relations on three-phase and n-phase sides are as follows.

$$V^n_{P'} = N U^3_P \tag{3.31}$$

$$U^3_P = \frac{1}{\sqrt{3}} P V^3_P \tag{3.32}$$

$$J^3_P = \frac{1}{K_m} N^t\, I^n_P \tag{3.33}$$

$$I^3_P = \frac{1}{\sqrt{3}\, K_m} P^t\, N^t\, I^n_P \tag{3.34}$$



3.4 A 3-phase wye / n-phase star transformer with off nominal turn ratio $\alpha_{3t} : \alpha_{nt}$.

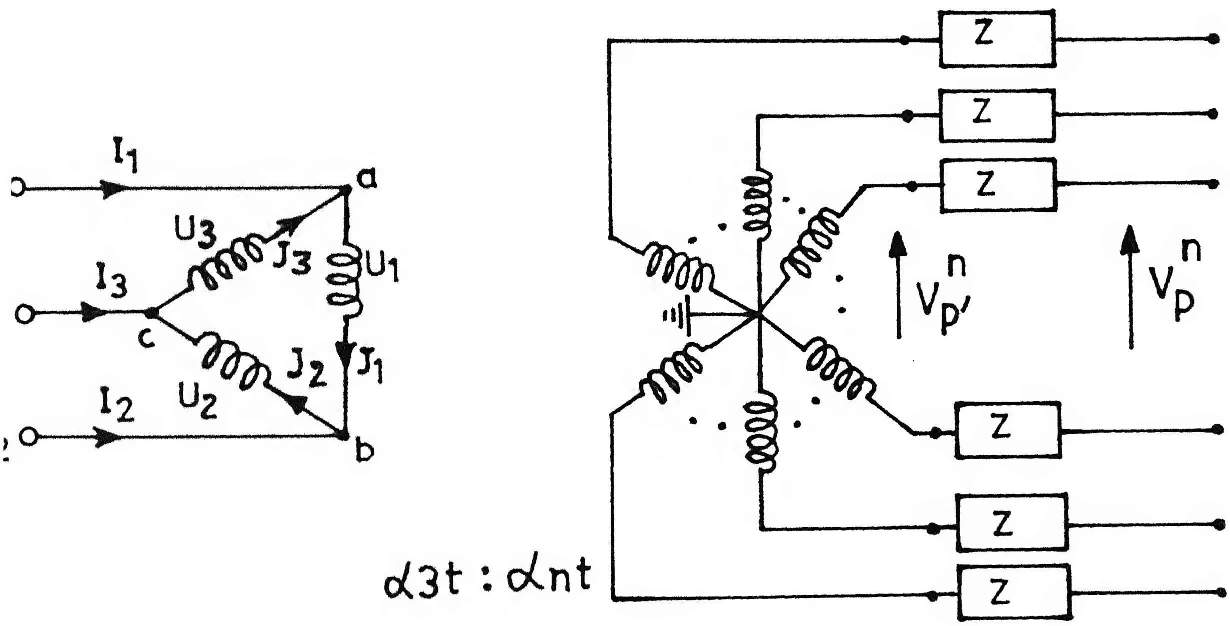


FIG. 3.5 A 3-phase Delta/n-phase star transformer with off-nominal turn ratio $\alpha_{3t} : \alpha_{nt}$.

where connecting matrix[P] = $$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \qquad (3.35)$$

From equations (3.31)-(3.34), the voltage and current relations are

$$V^n_{P'} = \frac{1}{\sqrt{3}}[N][P]\,V^3_P \quad ; \quad V^3_P = \frac{\sqrt{3}}{K_m}[P]^t\,[N]^t\,V^n_{P'} \qquad (3.36)$$

$$I^n_P = \sqrt{3}\,[N][P]\,I^3_P \; ; \; I^3_P = \frac{1}{\sqrt{3}\,K_m}\,[P]^t\,[N]^t\,I^n_P \qquad (3.37)$$

If the off-nominal turn ratio α_{3t} in the three-phase primary side is also taken into consideration, the equation (3.31)-(3.34) become

$$V^n_P = \frac{1}{\alpha_{3t}}\,[N]\,U^3_P \qquad (3.38)$$

$$U^3_P = \frac{1}{\sqrt{3}}[P]\,V^3_P \qquad (3.39)$$

$$J^3_P = \frac{1}{\alpha_{3t}\,K_m}[N]^t\,I^n_P \qquad (3.40)$$

$$I^3_P = \frac{1}{\sqrt{3}\alpha_{3t}}\,\frac{1}{K_m}\,.\,[P]^t\,[N]^t\,I^n_P \qquad (3.41)$$

The corresponding voltage and currents on the two sides are

$$V^n_{P'} = \frac{1}{\sqrt{3\alpha_{3t}}}[N][P]\,V^3_P \; ; \; V^3_P = \frac{\sqrt{3\alpha_{3t}}}{K_m}\,[P]^t[N]^t\,V^n_{P'} \qquad (3.42)$$

$$I^n_P = \sqrt{3}\,\alpha_{3t}\,[N][P]\,I^3_P \; ; \; I^3_P = \frac{1}{\sqrt{3\alpha_{3t}K_m}}[P]^t[N]^t\,I^n_P \qquad (3.43)$$

Now, if the tappings on both the sides of the transformer are considered, the equations (3.38)-(3.41) become

$$\frac{V^n_{P'}}{\alpha_{nt}} = \frac{1}{\sqrt{3\alpha_{3t}}} [N] \; U^3_P \tag{3.44}$$

$$U^3_P = \frac{1}{\sqrt{3}} [P] \; V^3_P \tag{3.45}$$

$$J^3_P = \frac{1}{\alpha_{3t} K_m} [N]^t \; I^n_P \tag{3.46}$$

$$I^3_P = \frac{1}{\sqrt{3\alpha_{3t}}} \; \frac{1}{K_m} [P]^t [N]^t \; I^n_P \tag{3.47}$$

The corresponding voltage and currents on the two sides are

$$V^n_{P'} = \frac{\alpha_{nt}}{\sqrt{3\alpha_{3t}}}[N][P] \, V^3_P \; ; \; V^3_P = \frac{\sqrt{3\alpha_{3t}}}{\alpha_{nt}} \cdot \frac{1}{K_m} [P]^t [N]^t \, V^n_{P'} \tag{3.48}$$

$$I^n_P = \frac{\sqrt{3\alpha_{3t}}}{\alpha_{nt}} [N][P] \, I^3_P \; ; I^3_P = \frac{\alpha_{nt}}{\sqrt{3\alpha_{3t}}} \cdot \frac{1}{K_m}[P]^t [N]^t \, I^n_P \tag{3.49}$$

The voltage and current relationship on the three-phase and n-phase sides of the transformer can be cast into a general format from the pairs of equations (3.29),(3.30) and (3.48), (3.49) as

$$V^n_{P'} = \frac{\alpha_{nt}}{\alpha'_{3t}}[H] \, V^3_P \; ; \quad V^3_P = \frac{\alpha'_{3t}}{\alpha_{nt}} \cdot \frac{1}{K_m}[H]^t \, V^n_{P'} \tag{3.50}$$

$$I^n_P = \frac{\alpha'_{3t}}{\alpha_{nt}}[H] \; I^3_P \; ; \quad I^3_P = \frac{\alpha_{nt}}{\alpha'_{3t}} \cdot \frac{1}{K_m}[H]^t \; I^n_P \tag{3.51}$$

where,

$\alpha'_{3t} = \alpha_{3t}$; $[H] = [N]$; for a three-phase Wye/n-phase star

and

$\alpha'_{3t} = \sqrt{3}\,\alpha_{3t}$; $[H] = [N][P]$ for a three-phase delta/n-phase star ; and α_{nt} represents the tapping on n-phase side of the transformer. Thus equations (3.50) and (3.51) represent in general the voltage and current relations of the two sides of the three-phase/n-phase transformer with the off-nominal turn ratios α'_{3t} and α_{nt}, where the first subscripts 3 and n denote the phase order. From these equations, such relations can be obtained for different cases under consideration. The pairs of equations (3.24)-(3.25), (3.27)-(3.28), (3.29)-(3.30), (3.36)-(3.37), (3.42)-(3.43), (3.48)-(3.49) are the particular cases with the different tappings and types of winding connections of the two sides of the transformers. For the different values of n, for example n = 6,12 , the relations can be obtained by substituting $K_m = 2,\ 3-j\sqrt{3}$ in equations (3.50) and (3.51).

3.3.2 Nodal Representation of the Transformer

For the purpose of mathematical modelling, the transformer has been assumed to be represented as an ideal one in series with the equivalent leakage impedance/admittance of the winding. The terminal voltages of the transformer in

terms of the induced voltage $V_{P'}^{n}$, can be expressed in generalized form from figures 3.4 and 3.5 as

$$V_P^n = V_{P'}^n - [Z_t^n] \quad I_P^n \tag{3.52}$$

where

$[Z_t^n]$ is a diagonal matrix of order (nxn) having impedances of the windings as the diagonal elements = $[Y_t^n]^{-1}$

From equations (3.50) and (3.52),

$$I_P^n = \left(\frac{\alpha_{nt}}{\alpha'_{3t}}\right) [Y_t^n][H] V_P^3 - [Y_t^n] V_P^n \tag{3.53}$$

and from equations (3.51) and (3.53)

$$I_P^3 = \left(\frac{\alpha_{nt}}{\alpha'_{3t}}\right)^2 \frac{1}{K_m} \cdot [H]^t [Y_t^n][H] V_P^3 - \left(\frac{\alpha_{nt}}{\alpha'_{3t}}\right) \frac{1}{K_m} \cdot [H]^t [Y_t^n] V_P^n \tag{3.54}$$

From equations (3.53) and (3.54), the nodal representation of the transformer is obtained as

$$\begin{bmatrix} I_P^3 \\ \\ -I_P^n \end{bmatrix} = \begin{bmatrix} \left(\frac{\alpha_{nt}}{\alpha'_{3t}}\right)^2 \cdot \frac{1}{K_m} \cdot [H]^t [Y_t^n][H] & -\frac{\alpha_{nt}}{\alpha'_{3t}} \frac{1}{K_m} [H]^t [Y_t^n] \\ \\ -\frac{\alpha_{nt}}{\alpha'_{3t}} [Y_t^n][H] & [Y_t^n] \end{bmatrix} \begin{bmatrix} V_P^3 \\ \\ V_P^n \end{bmatrix} \tag{3.55}$$

Equation (3.55) describes the nodal representation of a three-phase/n-phase transformer in generalized form rom which a particular case can be conveniently obtained by substituting the corresponding values.

3.4 MULTI-PHASE TRANSMISSION LINE MODEL

A three-phase transmission line is represented by phase impedance or admittance matrix, equivalent π-circuit representation and ABCD parameters [61]. The multi-phase transmission line in general can similary be described on the basis of three-phase line in terms of phase impedance matrix as,

$$[Z_p^n] = \begin{bmatrix} Z_{11} & Z_{12} & \cdot\,\cdot & Z_{1i} & \cdot\,\cdot & Z_{1n} \\ Z_{21} & Z_{22} & \cdot\,\cdot & Z_{2i} & \cdot\,\cdot & Z_{2n} \\ \cdot & \cdot & \cdot\,\cdot & \cdot & \cdot\,\cdot & \cdot \\ \cdot & \cdot & \cdot\,\cdot & \cdot & \cdot\,\cdot & \cdot \\ Z_{i1} & Z_{i2} & \cdot\,\cdot & Z_{ii} & \cdot\,\cdot & Z_{in} \\ \cdot & \cdot & \cdot\,\cdot & \cdot & \cdot\,\cdot & \cdot \\ \cdot & \cdot & \cdot\,\cdot & \cdot & \cdot\,\cdot & \cdot \\ Z_{n1} & Z_{n2} & & Z_{ni} & & Z_{nn} \end{bmatrix} \qquad (3.56)$$

where n is the number of phases.

If it is assumed that the multi-phase transmission line is completely transposed, (though it is very difficult to realize in practice) the impedance matrix $[Z_p^n]$ can be made diagonal matrix with the help of symmetrical and Clarke's component transformations, which can be developed on the basis of higher order transformation available in the literature [13-15,54] .

Whereas the short lines can be represented by equation (3.56), the representation of the medium multi-phase lines can be obtained by a straight forward extension of an equivalent π-circuit model of 3-phase line [61] and it is as follows.

$$\begin{bmatrix} I^n_{PS} \\ \\ I^n_{PR} \end{bmatrix} = \begin{bmatrix} Y^n_P + \frac{1}{2} Y^n_{sh} & -Y^n_P \\ \\ -Y^n_P & Y^n_P + \frac{1}{2} Y^n_{sh} \end{bmatrix} \begin{bmatrix} V^n_{PS} \\ \\ V^n_{PR} \end{bmatrix} \qquad (3.57)$$

where the second subscripts S and R denote the sending and the receiving ends of a multi-phase line respectively,

$$[Y^n_P] = [Z^n_P]^{-1} \qquad (3.58)$$

and

$$[Y^n_{sh}] = \begin{bmatrix} Y_{sh,11} & Y_{sh,12} & \cdot\cdot & Y_{sh,1i} & \cdot\cdot & Y_{sh,1n} \\ Y_{sh,21} & Y_{sh,22} & \cdot\cdot & Y_{sh,2i} & \cdot\cdot & Y_{sh,2n} \\ \vdots & \vdots & \vdots\vdots & \vdots & \vdots\vdots & \vdots \\ Y_{sh,i1} & Y_{sh,i2} & \cdot\cdot & Y_{sh,ii} & & Y_{sh,in} \\ \vdots & \vdots & \vdots\vdots & \vdots & \vdots\vdots & \vdots \\ Y_{sh,n1} & Y_{sh,n2} & \cdot\cdot & Y_{sh,ni} & \cdot\cdot & Y_{sh,nn} \end{bmatrix} \qquad (3.59)$$

where,

$Y_{sh,ij}$ is the shunt admittance between nodes i and j, if $i \neq j$,

$$\text{otherwise} \quad Y_{sh,ii} = \sum_{\substack{j=1 \\ \neq i}}^{n+1} Y_{sh,ij} \; ; \; i=1,2,..,n. \qquad (3.60)$$

In the present formulation, every phase (1-n) has been assigned a node number and (n+1)th node refers to ground.

The shunt admittances for multi-phase line is shown in Fig. 3.6.

Another useful representation of the transmission line in terms of ABCD parameters may be expressed as,

$$\begin{bmatrix} V_S^n \\ I_S^n \end{bmatrix} = \begin{bmatrix} A^n & B^n \\ C^n & D^n \end{bmatrix} \begin{bmatrix} V_R^n \\ I_R^n \end{bmatrix} \tag{3.61}$$

where A^n, B^n, C^n and D^n are the square matrices of the order (nxn).

For π -circuit representation,

$$A^n = D^n = \text{diagonal} \quad [1 + Z_P^1 \; Y_{sh}^1/2 \,..\, 1+Z_P^1 Y_{sh}^1/2] \tag{3.62}$$

$$B^n = \text{diag.}\,[\, Z_P^1 \;..\; Z_P^1 \,] \tag{3.63}$$

and

$$C^n = \text{diag.}\,[\, Y_{sh}^1 \; (1 + Z_P^1 \; Y_{sh}^1/4)\,..\,Y_{sh}^1(1+Z_P^1 Y_{sh}^1/4)] \tag{3.64}$$

For a long transmission line,

$$A^n = D^n = \cosh\gamma l \tag{3.65}$$

$$B^n = [\, Z_c \,]\, \sinh\gamma l \tag{3.66}$$

$$C^n = [\, Z_c]^{-1} \quad \sinh\gamma l \tag{3.67}$$

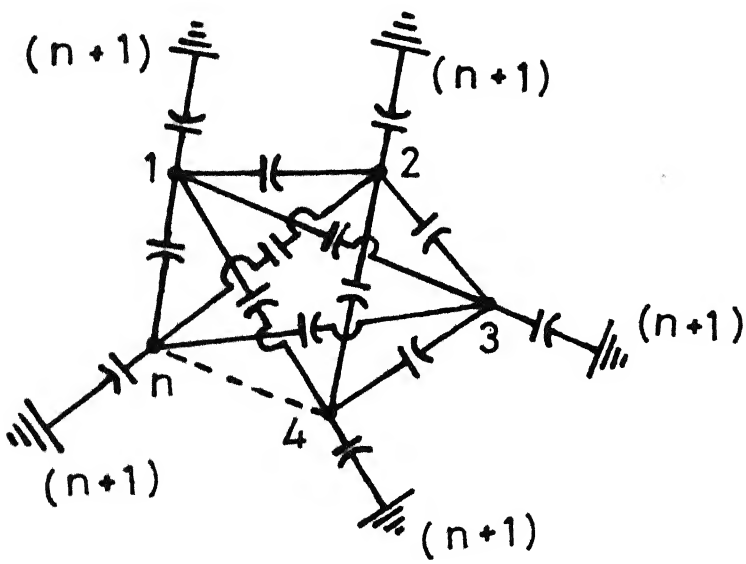


FIG.3.6 Equivalent capacitive susceptances at the terminating bus bars.

where,

$[Z_c]$, the characteristic impedance matrix $=\sqrt{Z/Y}$ and γ, propagation constant $=\sqrt{yZ}$; Z and y are the series impedance and the shunt admittance per unit length of line respectively.

Thus, equations (3.56), (3.57) and (3.61) represent the complete mathematical description of multi-phase transmission line in terms of phase impedance matrix, π-circuit representation and ABCD parameters respectively.

3.5 EQUIVALENT THREE-PHASE REPRESENTATION OF n-PHASE TRANS. LINE

In a composite system if the interest of investigation lies on the three-phase side of the system, it is sufficient to represent the n-phase transmission line along with a three-phase/n-phase transformer at the each end of the line by its equivalent three-phase parameters to carry out the analysis of the composite system. Equivalent three-phase representation of the n-phase transmission line has been derived in terms of phase impedances and also in terms of ABCD parameters under the following sections.

3.5.1 Equivalent Three-Phase Impedances of a Multi-Phase (n-Phase) Line

In order to derive the equivalent three-phase impedance, the multi-phase line has been assumed to be

connected between three-phase buses S and R through three-phase/n-phase transformers T_1 and T_2 as shown in Fig. 3.7. Two ideal transformers T_1 and T_2 are connected between S and S', and between R and R' with their leakage impedances connected in series between S' and S'', and between R' and R'' respectively. Let the n-phase transmission line connected between n-phase buses S'' and R'' be represented by

$$V^n_{S''} - V^n_{R''} = Z^n_p \; I^n_{p''} \tag{3.68}$$

Now the three-phase voltage at buses S and R can be obtained in terms of n-phase voltages and currents $V^n_{S''}$, $V^n_{R''}$ and $I^n_{p''}$ respectively by introducing the voltage relation of equations (3.50) in equation (3.52) corresponding to the two transformers connected at each end of transmission line, and which are as follows.

$$V^3_S = \frac{a'_{3t1}}{a_{nt1}} \frac{1}{K_m} [H]^t \, V^n_{S''} + \frac{a'_{3t1}}{a_{nt1}} \frac{1}{K_m} [H]^t [Z^n_{t1}] \, I^n_{S''} \tag{3.69}$$

and

$$V^3_R = \frac{a'_{3t2}}{a_{nt2}} \frac{1}{K_m} [H]^t \, V^n_{R''} - \frac{a'_{3t2}}{a_{nt2}} \frac{1}{K_m} [H]^t [Z^n_{t2}] \, I^n_{R''} \tag{3.70}$$

where

$[Z^n_{t1}]$ and $[Z^n_{t2}]$ are the diagonal matrices of order (nxn) having the leakage impedances of the transformers T_1 and T_2 respectively as their diagonal elements.

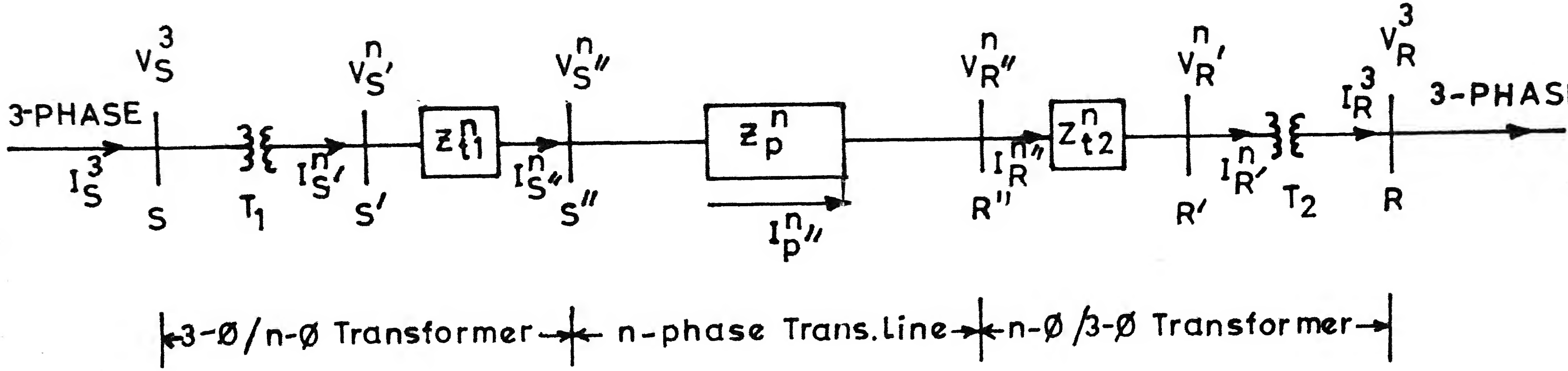


FIG. 3-7 A schematic Representation of a n-phase Transmission line connected to 3-phase buses S and R Via 3-phase/n-phase transformers.

Subtracting equation (3.70) from (3.69) and using the voltage relation of multi-phase transmission line in equation (3.68), the following equation is obtained.

$$\frac{\alpha_{nt1}}{\alpha_{3t1}} V_S^3 - \frac{\alpha_{nt2}}{\alpha_{3t2}} V_R^3 = \frac{1}{K_m}[H]^t[[Z_p^n]+[Z_{t1}^n]+[Z_{t2}^n]]I_{R''}^n \quad (3.71)$$

where $I_{P''}^n = I_{S''}^n = I_{R''}^n$

If the transformers T_1 and T_2 have nominal turn-ratios, equation (3.71) yields the three-phase equivalent impedance as follows

$$Z_{P,eq}^3 = \frac{1}{K_m}[H]^t\,[Z_P^n + Z_{t1}^n + Z_{t2}^n][H] \quad (3.72)$$

If the leakage impedances of the transformers are neglected, the equation (3.72) reduces to

$$Z_{P,eq}^3 = \frac{1}{K_m}[H]^t\,[Z_P^n]\,[H] \quad (3.73)$$

Equation (3.72) represents the three-phase equivalent of n-phase transmission line associated with two transformers at the ends with nominal turn-ratios.

If the n-phase transmission line is assumed to be completely transposed i.e. Z_P^n is an impedance matrix of order (nxn) whose diagonal terms are equal to Z_S and the off-diagonal terms are equal to Z_m, then from equation (3.72),

$$Z^3_{P,eq} = \begin{bmatrix} \frac{1}{K_m}(Z'_S K_m - Z_m K'_m) & 0 & 0 \\ 0 & \frac{1}{K_m}(Z'_S K_m - Z_m K'_m) & 0 \\ 0 & 0 & \frac{1}{K_m}(Z'_S K_m - Z_m K'_m) \end{bmatrix} \tag{3.74}$$

where, $Z'_S = Z_S + Z_1 + Z_2$

and

$$K'_m = 2\ [\ 1 + 1\angle{-m} + 1\angle{-2m} + 1\angle{-(\pi/3-m)}\ ] \tag{3.75}$$

Thus, the equivalent three-phase sequence impedances are

$$Z^{(o)}_{P,eq} = Z^{(1)}_{P,eq} = Z^{(2)}_{P,eq} = \frac{1}{K_m}(Z'_S K_m - Z_m K'_m) \tag{3.76}$$

Particular cases:

For a six-phase transmission line associated with the transformers at each end having nominal turn ratios, the equation (3.76) reduces to

$$Z^{(o)}_{P,eq} = Z^{(1)}_{P,eq} = Z^{(2)}_{P,eq} = Z'_S - Z_m \tag{3.77}$$

For a twelve-phase transmission line with the transformers having nominal turn-ratios, the sequence impedances are

$$Z^{(o)}_{P,eq} = Z^{(1)}_{P,eq} = Z^{(2)}_{P,eq} = Z'_S - \frac{K'_m}{K_m} \cdot Z_m \tag{3.78}$$

where, $$\frac{K'_m}{K_m} = \frac{1}{6}[\ (3+2\sqrt{3}) + j\sqrt{3}] \tag{3.79}$$

For a six-phase transmission line integrated with a three-phase Wye/six-phase star connected transformers, equations

(3.72) and (3.73) reduce to

$$Z^3_{P,eq} = \frac{1}{2} N^t [Z^6_P + Z^6_{t1} + Z^6_{t2}] N \tag{3.80}$$

and

$$Z^3_{P,eq} = \frac{1}{2} N^t Z^6_P N \tag{3.81}$$

For a twelve-phase transmission line integrated with a three-phase wye/twelve-phase star connected transformer at its each end, the equivalent three-phase impedance can be obtained by substituting $K_m = (3 - j\sqrt{3})$ in equations (3.72) and (3.73).

For a three-phase delta/six-phase star connected system the equation (3.72) and (3.73) reduce to

$$Z^3_{P,eq} = \frac{1}{2} [P]^t [N]^t [Z^6_P + Z^6_{t1} + Z^6_{t2}][N][P] \tag{3.82}$$

and

$$Z^3_{P,eq} = \frac{1}{2} [P]^t [N]^t [Z^6_P][N][P] \tag{3.83}$$

It is observed from the equation (3.71) that, if $a_{nt1}/a'_{3t1} \neq a_{nt2}/a'_{3t2}$, the three-phase equivalent parameters cannot be determined. However, if the two ratios are equal (the nominal turn-ratios of the transformers is a particular case), the equivalent three-phase parameters (phasor as well as sequence) can be determined. Moreover, it is observed that the connecting transformers increase the magnitude of parameters and from equation (3.76), it is further observed that

that the equivalent sequence impedances i.e. zero, positive and negative components are equal.

3.5.2 Equivalent Three-Phase ABCD Parameters of a Multi-Phase (n-Phase) Line

The n-phase transmission line forming a part of the three-phase power system connected at buses S'' and R'' via three-phase wye or Delta/n-phase star connected transformers T_1 and T_2 may be modelled by an equivalent ABCD parameters.

By taking the tappings on both the sides of the transformers T_1 are T_2 into considerations and using the equations (3.50) and (3.51), the voltages and currents at the sending ends (S,S' and S'') and the receiving ends (R,R',R'') can be related with the help of equations (3.52) and (3.68). From Fig. 3.7, the equation (3.52) can be expressed in the following form for sending ends.

$$V^n_{S'} = V^n_{S''} + [Z^n_{t1}] \; I^n_{S''} \tag{3.84}$$

From equations (3.50) and (3.84)

$$V^3_S = \frac{\alpha'_{3t1}}{\alpha_{nt1}} \; \frac{1}{K_m} [H]^t \; V^n_{S''} + \frac{\alpha'_{3t1}}{\alpha_{nt1}} \; \frac{1}{K_m} [H]^t [Z^n_{t1}] \, I^n_{S''} \tag{3.85}$$

and from the equation (3.51), we get

$$I^3_S = \frac{\alpha_{nt1}}{\alpha'_{3t1}} \cdot \frac{1}{K_m} [H]^t \; I^n_{S''} \tag{3.86}$$

Similary, the voltage and current relations at the receiving ends R,R' and R" can be expressed as

$$V_R^{n''} = \frac{\alpha_{nt2}}{\alpha'_{3t2}} [H] \; V_R^3 + \frac{\alpha'_{3t2}}{\alpha_{nt2}} [H][Z_{t2}^n] \; I_R^3 \tag{3.87}$$

and

$$I_{R''}^n = \frac{\alpha'_{3t2}}{\alpha_{nt2}} [H] \; I_R^3 \tag{3.88}$$

From the pairs of equations (3.85), (3.86) and (3.87), (3.88), the following representations in the matrix form are obtained.

$$\begin{bmatrix} V_S^3 \\ \\ I_S^3 \end{bmatrix} = \begin{bmatrix} \dfrac{\alpha'_{3t1}}{\alpha_{nt1}} \cdot \dfrac{1}{K_m} [H]^t & \dfrac{\alpha'_{3t1}}{\alpha_{nt1}} \cdot \dfrac{1}{K_m} [H]^t [Z_{t1}^n] \\ \\ 0 & \dfrac{\alpha_{nt1}}{\alpha'_{3t1}} \dfrac{1}{K_m} [H]^t \end{bmatrix} \begin{bmatrix} V_{S''}^n \\ \\ I_{S''}^n \end{bmatrix} \tag{3.89}$$

and

$$\begin{bmatrix} V_{R''}^n \\ \\ I_{R''}^n \end{bmatrix} \quad \begin{bmatrix} \dfrac{\alpha_{nt2}}{\alpha'_{3t2}} [H] & \dfrac{\alpha'_{3t2}}{\alpha_{nt2}} [H][Z_{t2}^n] \\ \\ 0 & \dfrac{\alpha'_{3t2}}{\alpha_{nt2}} \end{bmatrix} \begin{bmatrix} V_R^3 \\ \\ I_R^3 \end{bmatrix} \tag{3.90}$$

From equations (3.61), (3.89), (3.90), the equivalent three-phase representation in ABCD parameters is

$$\begin{bmatrix} V_S^3 \\ \\ I_S^3 \end{bmatrix} = \begin{bmatrix} A_{eq}^3 & B_{eq}^3 \\ \\ C_{eq}^3 & D_{eq}^3 \end{bmatrix} \begin{bmatrix} V_R^3 \\ \\ I_R^3 \end{bmatrix} \tag{3.91}$$

where,

$$A_{eq}^3 = \frac{\alpha'}{\alpha''} \cdot \frac{1}{K_m} [H]^t \, [A^n + Z_{t1}^n \; C^n] [H] \tag{3.92}$$

$$B_{eq}^3 = \alpha'\alpha'' \; \frac{1}{K_m} [H]^t [\; (\; A^n + [Z_{t1}^n] \; C^n \;) \; Z_{t2}^n + B^n + Z_{t1}^n \; D^n] [H] \tag{3.93}$$

$$C_{eq}^3 = \frac{1}{\alpha' \alpha''} \frac{1}{K_m} [H]^t \; C^n [\, H] \tag{3.94}$$

$$D_{eq}^3 = \frac{\alpha''}{\alpha'} \; \frac{1}{K_m} [H]^t \, [D^n + C^n \; Z_{t2}^n] [H] \tag{3.95}$$

and

$$\alpha' = \frac{\alpha'_{3t1}}{\alpha_{nt1}} \quad \text{and} \quad \alpha'' = \frac{\alpha'_{3t2}}{\alpha_{nt2}} \tag{3.96}$$

When the leakage impedances of the transformers T_1 and T_2 are neglected, the following equivalent three-phase parameters are obtained

$$A_{eq}^3 = \frac{\alpha'}{\alpha''} \; \frac{1}{K_m} [H]^t \; A^n \; [H] \tag{3.97}$$

$$B_{eq}^3 = \alpha'\alpha'' \frac{1}{K_m} [H]^t \; B^n \; [H] \tag{3.98}$$

$$C_{eq}^{3} = \frac{1}{\alpha' \alpha''} \frac{1}{K_m} [H]^t C^n [H] \tag{3.99}$$

$$D_{eq}^{3} = \frac{\alpha''}{\alpha'} \frac{1}{K_m} [H]^t D^n [H] \tag{3.100}$$

Equations (3.92)-(3.95) give the equivalent three-phase ABCD parameters in general. It is important to observe that the equivalent three-phase ABCD parameters can also be obtained by considering off-nominal turn ratios of the transformers, expressing the versality of this representation. Further, it is seen that A_{eq}^{3} and D_{eq}^{3} are not equal due to the off-nominal turn ratio and the different leakage impedances of the transformers.

If the tapping on the primary side of transformer T_1 is only considered, $\alpha' = \alpha'_{3t1}$ and $\alpha'' = 1$ can be substituted in the equations (3.92)-(3.96) to obtain the parameters.

3.6 EQUIVALENT SINGLE-PHASE REPRESENTATION OF A MULTI-PHASE (n-PHASE) TRANSMISSION LINE

The single-phase equivalent of a n-phase line for the analysis of a balanced system can be conveniently derived from the equivalent three-phase parameters in equations (3.92)-(3.95). Here, A_{eq}^{3}, B_{eq}^{3}, C_{eq}^{3} and D_{eq}^{3} are the square diagonal matrices with A_{eq}^{1}, B_{eq}^{1}, C_{eq}^{1} and D_{eq}^{1} as the diagonal entries respectively.

Let $Z^1_{P,eq}$ and $Y^1_{sh,eq}$ be the equivalent single-phase series impedance and shunt admittance of a n-phase line respectively, and Z^1_P, Y^1_{sh} be the series impedance and shunt admittance per phase of n-phase line, then the π-representation holds the following relations.

$$A^1_{eq} = 1 + \frac{1}{2} Z^1_{P,eq} Y^1_{sh,eq} \tag{3.101}$$

and $$B^1_{eq} = Z^1_{P,eq} \tag{3.102}$$

The values of A^1_{eq} and B^1_{eq} are evaluated from equations (3.92) and (3.93) and are as follows.

$$A^1_{eq} = \frac{\alpha'}{\alpha''} \left[1+\frac{1}{2} Z^1_P Y^1_{sh} + Z_1 Y^1_{sh}(1+0.25 Z^1_P Y^1_{sh}) \right] \tag{3.103}$$

$$B^1_{eq} = \alpha'\alpha'' \left[Z^1_P + (Z_1+Z_2)(1+\tfrac{1}{2} Z^1_P Y^1_{sh}) + Z_1 Z_2 Y^1_{sh}(1+0.25 Z^1_P Y^1_{sh}) \right] \tag{3.104}$$

From equations (3.101)-(3.104), Z^1_P and Y^1_{sh} can be related to $Z^1_{P,eq}$ and $Y^1_{sh,eq}$ by the following relations.

$$Z^1_{P,eq} = \alpha'\alpha'' \left[Z^1_P + (Z_1+Z_2)(1+\tfrac{1}{2} Z^1_P Y^1_{sh}) + Z_1 Z_2 Y^1_{sh}(1+0.25 Z^1_P Y^1_{sh}) \right] \tag{3.105}$$

and

$$Y^1_{sh,eq} = \frac{2}{Z^1_{P,eq}} \left[A^1_{eq} - 1 \right] \tag{3.106}$$

Thus equations (3.105) and (3.106) give in general the single-phase equivalent of the n-phase transmission line in terms of π-circuit representation.

If the transformers T_1 and T_2 are considered with nominal turn-ratios, the equations (3.105) and (3.106) reduce to

$$Z^1_{P,eq} = Z^1_P + (Z_1+Z_2)(1+0.5Z^1_P Y^1_{sh}) + Z_1 Z_2 Y^1_{sh}(1+0.25Z^1_P Y^1_{sh}) \quad (3.106a)$$

and

$$Y^1_{sh,eq} = \frac{1}{Z^1_{P,eq}} \left[Z^1_P Y^1_{sh} + (Z_1+Z_2) Y^1_{sh}(1+0.25Z^1_P Y^1_{sh}) \right] \quad (3.106b)$$

It is important to mention that the equivalent single-phase π-circuit parameters have been obtained using A^3_{eq} and B^3_{eq} in equations (3.92) and (3.93). But there are 4 equations of equivalent three-phase parameters, and any two equations can yield the desired single-phase π-circuit parameters from them.

Again, using equations (3.93) and (3.94) for π-representation, the following relations are obtained.

$$C^1_{eq} = Y^1_{sh,eq}(1+0.25\ Z^1_{P,eq}\ Y^1_{sh,eq}) \quad (3.106c)$$

and

$$B^1_{eq} = Z^1_{P,eq} \text{ as in equation (3.102)} \quad (3.106d)$$

From equation (3.102) and (3.106d), equation (3.106a) is obtained from equation (3.94), we get

$$C^1_{eq} = Y^1_{sh}(1+0.25Z^1_P Y^1_{sh}) \quad (3.106e)$$

From equations (3.106c) and (3.107e), the following quadratic equation is obtained.

$$\frac{Z^1_{P,eq}}{4}((Y^1_{sh,eq})^2+Y^1_{sh,eq}-Y^1_{sh}(1+0.25Z^1_PY^1_{sh}) = 0 \tag{3.106f}$$

which yields

$$Y^1_{sh,eq} = \frac{-1+\sqrt{[1+Z^1_{P,eq}(1+0.25Z^1_PY^1_{sh})Y^1_{sh}]}}{(Z^1_{P,eq}/2)} \tag{3.106g}$$

using Binomial expansion and neglecting the 3rd and the higher order terms, we get

$$Y^1_{sh,eq} = [\tfrac{1}{2}\, Z^1_{P,eq}Y^1_{sh}(1+0.25Z^1_PY^1_{sh})] \;/(Z^1_{P,eq}/2)$$

$$\cong Y^1_{sh}(1+0.25Z^1_PY^1_{sh}) \tag{3.196j}$$

Equation (3.106j) can also be expressed as

$$Y^1_{sh,eq} = \frac{1}{Z^1_{P,eq}}\;[Z^1_PY^1_{sh}+(Z_1+Z_2)Y^1_{sh}(1+0.25Z^1_PY^1_{sh})+Z_1Z_2(Y^1_{sh})^2$$

$$(1+0.25Z^1_PY^1_{sh})+(Z_1+Z_2)X.5\;Z^1_P(Y^1_{sh})^2(1+0.25Z^1_PY^1_{sh})] \tag{3.106k}$$

If the π-circuit parameters are derived from equation (3.92) and (3.94) the single-phase equivalent parameters will be

$$Z^1_{P,eq} = Z^1_P + 2Z_1(1+0.5Z^1_PY^1_{sh})+Z^2_1Y^1_{sh}(1+0.25Z^1_PY^1_{sh}) \tag{3.106l}$$

and

$$Y^1_{sh,eq} = \frac{1}{Z^1_{P,eq}} [Z^1_P Y^1_{sh} + 2Z_1 Y^1_{sh}(1+0.5Z^1_P Y^1_{sh}) + Z_1 (Y^1_{sh})^2 (1+0.25Z^1_P Y^1_{sh})$$

$$+.25Z^1_P Y^1_{sh} (Z^1_P Y^1_{sh} + 2Z_1 Y^1_{sh}(1+0.5Z^1_P Y^1_{sh}) + Z^2_1 (Y^1_{sh})^2 (1+0.25$$

$$Z^1_P Y^1_{sh}))] \tag{3.106m}$$

$$\cong Y^1_{sh}(1+0.25Z^1_P Y^1_{sh}) \tag{3.106n}$$

If the corresponding parameters are derived from equation (3.95) and (3.96), these can be obtained by substituting $Z_1 = Z_2$ in equations (3.106l) and (3.106m).

The pairs of equations (3.106a)-(3.106b), (3.106a) and (3.106j) or (3.106k), and (3.106l) and (3.106m) or (3.106n) give the equivalent single-phase π-representation of a balanced n-phase transmission line with nominal turn ratios. It is observed from the equivalent single-phase impedance in equations (3.106a) and (3.106l) that the value differs only due to difference in the leakage impedances of the two transformers. If the leakage impedances Z_1 and Z_2 are taken as equal, $Z^1_{P,eq}$ will be the same by all the pairs of equations considered.

From equations (3.106b), (3.106k) and (3.106m), it is observed that there is a difference in the calculated value of $Y^1_{sh,eq}$ from the different pairs of equations, but

from the practical point of view, the equation (3.106b) can be considered without appreciable error. The equivalent admittance can also be calculated from equation (3.106j) or (3.106n).

If the transformers T_1 and T_2 having equal leakage impedance are assumed to operate with nominal turn ratio, the equations (3.106a) and (3.106b) yield

$$Z^1_{P,eq} = Z^1_P + 2Z(1+0.5Z^1_P Y^1_{sh}) + Z^2 Y^1_{sh}(1+0.25Z^1_P Y^1_{sh}) \qquad (3.107)$$

and

$$Y^1_{sh,eq} = \frac{1}{Z^1_{P,eq}} \left[Z^1_P Y^1_{sh} + 2ZY^1_{sh}(1+0.25Z^1_P Y^1_{sh}) \right] \qquad (3.108)$$

3.7 EQUIVALENT n-PHASE REPRESENTATION OF A THREE-PHASE TRANSMISSION LINE

If the analysis is carried out on n-phase basis of the composite system, it is necessary to derive the equivalent n-phase parameters of the three-phase transmission line integrated with n-phase/three-phase transformers at the each end of the line. Thus, the equivalent n-phase of the line has been derived in terms of phase impedance parameters and also, in terms of ABCD parameters.

3.7.1 Equivalent n-Phase Impedances of a Three-Phase line

In order to derive the equivalent n-phase impedance, let us consider a three-phase line connected to the n-phase buses S and R via n-phase/three-phase transformers T_1 and T_2 as shown in Fig. 3.8.

From the Fig. 3.8, the following relations are obtained.

$$V_S^3 = V_{S'}^n + [Z_{t1}^n]\, I_{S'}^n$$

using equation (3.51), we get

$$V_S^n = \frac{\alpha_{nt1}}{\alpha'_{3t1}} [H]\, V_{S''}^3 + [Z_{t1}^n]\, I_{S''}^n \tag{3.109}$$

and

$$V_R^n = V_{R'}^n - [Z_{t2}^n]\, I_{R'}^n$$

$$= \frac{\alpha_{nt2}}{\alpha'_{3t2}} [H]\, V_{R''}^3 - [Z_{t2}^n]\, I_{R''}^n \tag{3.110}$$

Subtracting equation (3.110) from (3.109) and using the description of three-phase transmission line as

$$V_{S''}^3 - V_{R''}^3 = [Z_P^3]\, I_{P''}^3 \tag{3.111}$$

From equations (3.109) and (3.110), and using equation (3.111), we get

$$\frac{\alpha'_{3t1}}{\alpha_{nt1}} V_S^n - \frac{\alpha'_{3t2}}{\alpha_{nt2}} V_R^n = [H][Z_P^3]\, I_{P''}^3 + \left[\frac{\alpha'_{3t1}}{\alpha_{nt1}} \cdot [Z_{t1}^n] + \frac{\alpha'_{3t2}}{\alpha_{nt2}} \cdot [Z_{t2}^n]\right] I_{P''}^n \tag{3.112}$$

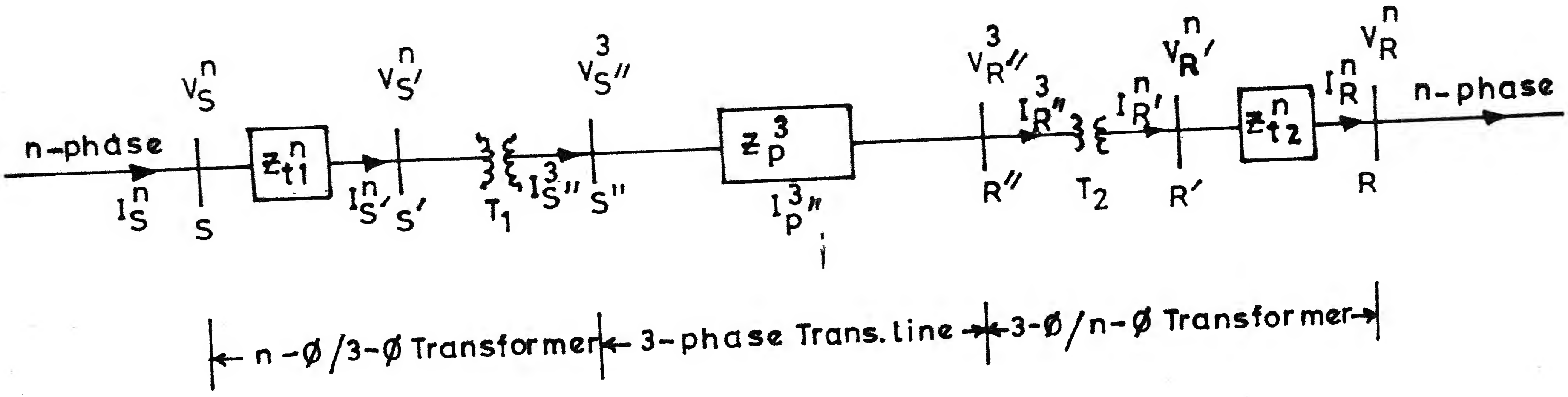


FIG.3.8 A schematic Representation of a three-phase line connected to n-phase buses S and R Via 3-phase/n-phase transformers.

Assuming nominal turn-ratio of the transformer T_1 and T_2, the following expression is obtained.

$$Z^n_{P,eq} = \frac{1}{K_m} \cdot [H][Z^3_p][H]^t + [Z^n_{t1} + Z^n_{t2}] \qquad (3.113)$$

Equation (3.113) represents the equivalent n-phase impedance of the three-phase transmission line.

3.7.2 Equivalent n-phase ABCD Parameters of a Three-Phase Line

The equivalent n-phase ABCD parameters of a three-phase transmission line can be derived as follows. Let us consider a three-phase line connected between two n-phase buses S and R via two three-phase/n-phase transformers as shown in Fig. 3.8.

Then, the following relations can be obtained from the Fig.3.8.

$$V^n_S = V^n_{S'} + [Z^n_{t1}]\, I^n_{S'}$$

using equations (3.50) and (3.51),

$$V^n_S = \frac{\alpha_{nt1}}{\alpha'_{3t1}}[H]V^3_{S''} + [Z^n_{t1}]\frac{\alpha'_{3t1}}{\alpha_{nt1}}[H]\, I^3_{S''} \qquad (3.114)$$

and

$$I^n_S = \frac{\alpha'_{3t1}}{\alpha_{nt1}}[H]\, I^3_{S''} \qquad (3.115)$$

From equations (3.114) and (3.115), and using equation (3.96),

$$\begin{bmatrix} V_S^n \\ \\ I_S^n \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha'}[H] & \alpha'[Z_{t1}^n][H] \\ \\ 0 & \alpha'[H] \end{bmatrix} \begin{bmatrix} V_{S''}^3 \\ \\ I_{S''}^3 \end{bmatrix} \tag{3.116}$$

Similarly, the voltage and current relations at R,R' and R''

are

$$V_{R'}^n = V_R^n + [Z_{t2}^n] I_R^n$$

or

$$\frac{\alpha_{nt2}}{\alpha'_{3t2}}[H]\, V_{R''}^3 = V_R^n + [Z_{t2}^n]\; I_R^n$$

$$V_{R''}^3 = \frac{\alpha'_{3t2}}{\alpha_{nt2}} \frac{1}{K_m}[H]^t\; V_R^n + \frac{\alpha'_{3t2}}{\alpha_{nt2}} \cdot \frac{1}{K_m}[H]^t[Z_{t2}^n]\, I_R^n \tag{3.117}$$

and

$$I_{R''}^3 = \frac{\alpha_{nt2}}{\alpha'_{3t2}} \frac{1}{K_m}[H]^t\; I_R^n \tag{3.118}$$

From equations (3.117) and (3.118)

$$\begin{bmatrix} V_{R''}^3 \\ \\ I_{R''}^3 \end{bmatrix} = \begin{bmatrix} \frac{\alpha''}{K_m}[H]^t & \frac{\alpha''}{K_m}[H]^t\,[Z_{t2}^n] \\ \\ 0 & \frac{\alpha''}{K_m}[H]^t \end{bmatrix} \begin{bmatrix} V_R^n \\ \\ I_R^n \end{bmatrix} \tag{3.119}$$

The three-phase line connected between S" and R" can be modelled as

$$\begin{bmatrix} V^3_{S''} \\ \\ I^3_{S''} \end{bmatrix} = \begin{bmatrix} A^3 & B^3 \\ \\ C^3 & D^3 \end{bmatrix} \begin{bmatrix} V^3_{R''} \\ \\ I^3_{R''} \end{bmatrix} \tag{3.120}$$

From equation (3.116), (3.119) and (3.120), the following n-phase equivalent is obtained.

$$\begin{bmatrix} V^n_{S} \\ \\ I^n_{S} \end{bmatrix} = \begin{bmatrix} A^n_{eq} & B^n_{eq} \\ \\ C^n_{eq} & D^n_{eq} \end{bmatrix} \begin{bmatrix} V^n_{R} \\ \\ I^n_{R} \end{bmatrix} \tag{3.121}$$

where,

$$A^n_{eq} = \frac{1}{K_m}\left[\frac{\alpha''}{\alpha'} [H] A^3 [H]^t + \alpha'\alpha'' [Z^n_{t1}][H]C^3[H]^t\right] \tag{3.122}$$

$$B^n_{eq} = \frac{1}{K_m}\left[\frac{\alpha''}{\alpha'} [H] A^3 [H]^t [Z^n_{t2}] + \alpha'\alpha'' [Z^n_{t1}][H]C^3[H]^t [Z^n_{t2}] + \frac{1}{\alpha'\alpha''} [H]B^3[H]^t + \frac{\alpha'}{\alpha''} [Z^n_{t1}][H]D^3[H]^t\right] \tag{3.123}$$

$$C^n_{eq} = \frac{1}{K_m} \alpha'\alpha'' [H]C^3[H]^t \tag{3.124}$$

$$D^n_{eq} = \frac{1}{K_m}\left[\frac{\alpha'}{\alpha''} [H] D^3 [H]^t + \alpha'\alpha'' [H] C^3[H]^t [Z^n_{t2}]\right] \tag{3.125}$$

Thus, equations (3.122) - (3.125) give the equivalent n-phase ABCD parameters of the three-phase line.

3.8 CONCLUSION

The mathematical model of a general n-phase (multi-phase) synchronous machine has been developed in phasor co-ordinates that will help for its steady-state analysis both under balanced as well as under unbalanced conditions. Further, three-phase/n-phase transformers having off-nominal turn ratio on both the sides have been modelled in order to analyse the combined three-phase and multi-phase systems by considering two types of three-phase windings, namely, wye and delta, and one type of multi-phase windings i.e., star connected winding. Further, the multi-phase transmission line has been represented in terms of phase impedances, π-circuit admittance, and ABCD parameters that will help in analysing the short, medium and long lines respectively.

The three-phase equivalent of a n-phase transmission line integrated with three-phase/n-phase transformers has been obtained in order to carry out the analysis on a composite system because of less computational efforts, when the interest of investigation lies on the three-phase side of the system. For the analysis of a balanced composite system in order to carry out the investigation on single-phase basis, the equivalent single-phase parameters representing π-circuit have been obtained based on the three-phase equivalent ABCD parameters. The equivalent n-phase parameter for the three-phase line

has also been developed in terms of impedances and ABCD parameters from which the single-phase equivalent can conveniently be derived. This equivalent n-phase parameters will be needed when the three-phase line is connected to n-phase lines by means of three-phase to n-phase power transformers and the interest of investigation lies in the n-phase line of the composite system.

CHAPTER 4

LOAD FLOW ANALYSIS OF SIX-PHASE (MULTI-PHASE) POWER SYSTEM NETWORKS

4.1 INTRODUCTION

In the previous chapter, mathematical discriptions of different components of power system networks have been derived. This chapter presents the load flow analysis of a combined three-phase and multi-phase systems with the help of the equivalent single-phase parameters of a multi-phase transmission line connected to three-phase/n-phase power transformers assuming the power system to be completely balanced, in order to compare the performance of a multi-phase transmission line relative to that of the conventional three-phase system.

There is a vast literature available for a three-phase power system load flow analysis [25, 62-67]. With the emergence of the idea of multi-phase transmission line, a load flow study has been carried out on mixed three-phase and six-phase power systems on the single-phase basis [2,21] , and also in the phase frame of reference [30] in order to compare the performance of the mixed system to that of completely three-phase system having double circuit three-phase lines. In the analysis, the phase to ground voltage of a six-phase line has been assumed to be equal to $\sqrt{3}$ times that of the three-phase line. This chapter is also an attempt in this

direction by presenting the load flow investigation for the different situations under different voltage conditions namely, the phase to ground voltage of the multi-phase line is $\sqrt{3}$ times or equal to that of a double circuit three-phase line, when a double circuit three-phase line is converted to a single circuit multi-phase line.

The conversion of a double circuit three-phase line to single-circuit multi-phase line **will** essentially require a three-phase/n-phase power transformer at each end of the multi-phase line. In the cases, as mentioned above, the conversion requires for three-phase/n-phase transformers, the varying voltage on the n-phase side ($\sqrt{3}$ times or equal to that of the double circuit three-phase line). Therefore, in order to analyse such a balanced system under different voltages of three-phase and multi-phase lines, the equivalent single-phase parameters of a multi-phase line integrated with two three-phase/n-phase transformers at the ends incorporating the different voltages on n-phase side have been developed in section 4.2. Different types of problems have been formulated for the investigation of load flow analysis in section 4.3 along with the discussion of the load flow methods required for the study. Different cases for the investigation have been taken on two sample networks in section 4.4. In section 4.4.1, a single machine 3-Bus system has been taken to

study the load flow performance for the different cases in order to compare the performance of systems, namely a mixed three-phase and multi-phase transmission system and a completely three-phase transmission systems; a multi-phase transmission line is obtained by replacing a double circuit three-phase line.

The same problems of load flow analysis have been investigated in section 4.4.2 on a three-machine nine-Bus power system network by replacing three double circuit three-phase lines to single circuit multi-phase lines one by one. The results of the investigation on two sample networks for different cases have been presented and discussed in section 4.5. Finally, the chapter ends in section 4.6 with the conclusions based on the discussions of the results.

4.2 EQUIVALENT SINGLE-PHASE REPRESENTATION

This section derives the equivalent single-phase parameters for π-representation of a multi-phase transmission line integrated with three-phase/n-phase transformers at the ends of the line to facilitate the analysis on a single-phase basis of the combined three-phase and multi-phase power system networks. With the help of a three-phase/n-phase transformer, the power can be transmitted through multi-phase transmission line at the varying voltages, and therefore, the expressions for equivalent single-phase series impedance and

shunt admittance have been derived for general case including the number of phases and the different voltages on n-phase side of the transformer.

A multi-phase transmission line is connected to three-phase buses through three-phase wye/n-phase star connected transformers as shown in Fig.3.7.

From Fig. 3.7, the voltages at the sending end can be expressed as,

$$V^n_{S'} = V^n_{S''} + [Z^n_{t1}] \quad I^n_{S''} \tag{4.1}$$

$$\frac{\alpha_{nt1}}{\alpha_{3t1}} [N] V^3_S = V^n_{S''} + [Z^n_{t1}] I^n_{S''}$$

$$V^3_S = \frac{1}{K_m} \frac{\alpha_{3t1}}{\alpha_{nt1}} [N]^t \quad V^n_{S''} + \frac{1}{K_m} \frac{\alpha_{3t1}}{\alpha_{nt1}} [N]^t [Z^n_{t1}] \quad I^n_{S''} \tag{4.2}$$

$$I^3_S = \frac{n}{3} \times \frac{\alpha_{nt1}}{\alpha_{3t1}} \cdot \frac{1}{K_m} [N]^t \quad I^n_{S''} \tag{4.3}$$

Similary, the voltages at the receiving end are related as follows.

$$V^n_{R''} = V^n_{R'} + [Z^n_{t2}] I^n_{R''}$$

$$= \frac{\alpha_{nt2}}{\alpha_{3t2}} [N] V^3_R + [Z^n_{t2}] \frac{3}{n} \frac{\alpha_{3t2}}{\alpha_{nt2}} [N] I^3_R \tag{4.4}$$

$$I^n_{R''} = \frac{3}{n} \cdot \frac{\alpha_{3t2}}{\alpha_{nt2}} [N] \quad I^3_R \tag{4.5}$$

From equations (4.2)-(4.3) and (4.4-4.5), the following representations in matrix form are obtained.

$$\begin{bmatrix} V_S^3 \\ \\ I_S^3 \end{bmatrix} = \begin{bmatrix} \frac{1}{K_m} \frac{\alpha_{3t1}}{\alpha_{nt1}} [N]^t & \frac{1}{K_m} \frac{\alpha_{3t1}}{\alpha_{nt1}} [N]^t [Z_{t1}^n] \\ \\ 0 & \frac{n}{3} \frac{1}{K_m} \frac{\alpha_{nt1}}{\alpha_{3t1}} [N]^t \end{bmatrix} \begin{bmatrix} V_{S''}^n \\ \\ I_{S''}^n \end{bmatrix} \tag{4.6}$$

and

$$\begin{bmatrix} V_{R''}^n \\ \\ I_{R''}^n \end{bmatrix} = \begin{bmatrix} \frac{\alpha_{nt2}}{\alpha_{3t2}} [N] & \frac{3}{n} \frac{\alpha_{3t2}}{\alpha_{nt2}} [Z_{t2}^n] [N] \\ \\ 0 & \frac{\alpha_{3t2}}{\alpha_{nt2}} [N] \end{bmatrix} \begin{bmatrix} V_R^3 \\ \\ I_R^3 \end{bmatrix} \tag{4.7}$$

From equations (3.61), (4.6) and (4.7), we get,

$$\begin{bmatrix} V_S^3 \\ \\ I_S^3 \end{bmatrix} = \begin{bmatrix} A_{eq}^3 & B_{eq}^3 \\ \\ C_{eq}^3 & D_{eq}^3 \end{bmatrix} \begin{bmatrix} V_R^3 \\ \\ I_R^3 \end{bmatrix} \tag{4.8}$$

where

$$A_{eq}^3 = \frac{1}{K_m} \frac{\alpha_{3t1}}{\alpha_{nt1}} \cdot \frac{\alpha_{nt2}}{\alpha_{3t2}} [N]^t [A^n + [Z_{t1}^n]] C^n] [N] \tag{4.9a}$$

$$B_{eq}^3 = \frac{3}{n} \frac{1}{K_m} \frac{\alpha_{3t1}}{\alpha_{nt1}} \cdot \frac{\alpha_{3t2}}{\alpha_{nt2}} \cdot N^t [A^n + [Z_{t1}^n] C^n] [Z_{t2}^n] + B^n + [Z_{t2}^n] D^n] [N] \tag{4.9b}$$

$$C_{eq}^3 = \frac{n}{3} \frac{1}{K_m} \frac{\alpha_{nt1}}{\alpha_{3t1}} \cdot \frac{\alpha_{nt2}}{\alpha_{3t2}} [N]^t C^n [N] \tag{4.9c}$$

$$D^3_{eq} = \frac{1}{K_m} \cdot \frac{\alpha_{3t2}}{\alpha_{nt2}} \cdot \frac{\alpha_{nt1}}{\alpha_{3t1}} [N]^t \, [D^n + C^n [Z^n_{t2}]] \, N \tag{4.9d}$$

A^3_{eq}, B^3_{eq}, C^3_{eq} and D^3_{eq} are the diagonal matrices which have A^1_{eq}, B^1_{eq}, C^1_{eq} and D^1_{eq} as their diagonal elements respectively. From equations (4.9a)-(4.9d), the respective values are

$$A^1_{eq} = \frac{\alpha_{3t1}}{\alpha_{nt1}} \cdot \frac{\alpha_{nt2}}{\alpha_{3t2}} [\, 1+\frac{1}{2} Z^1_p Y^1_{sh} + Z_1 Y^1_{sh} (1+0.25 Z^1_p Y^1_{sh}) \,] \tag{4.10a}$$

$$B^1_{eq} = \frac{3}{n} \frac{\alpha_{3t1}}{\alpha_{nt1}} \cdot \frac{\alpha_{3t2}}{\alpha_{nt2}} [\, Z^1_p + (Z_1+Z_2)(1+\frac{1}{2} Z^1_p Y^1_{sh}) + Z_1 Z_2 Y^1_{sh} (1+0.25 Z^1_p Y^1_{sh}) \,] \tag{4.10b}$$

$$C^1_{eq} = \frac{n}{3} \frac{\alpha_{nt1}}{\alpha_{3t1}} \cdot \frac{\alpha_{nt2}}{\alpha_{3t2}} Y^1_{sh} \, (1+\frac{1}{4} Z^1_p Y^1_{sh}) \tag{4.10c}$$

$$D^1_{eq} = \frac{\alpha_{3t2}}{\alpha_{nt2}} \cdot \frac{\alpha_{nt1}}{\alpha_{3t1}} [\, 1+\frac{1}{2} Z^1_p Y^1_{sh} + Z_2 Y^1_{sh} \, (1+0.25 \, Z^1_p Y^1_{sh}) \,] \tag{4.10d}$$

where, Z^1_p and Y^1_{sh} are the series impedance and the shunt admittance of a multi-phase transmission line, and Z_1 and Z_2 are the diagonal elements of diagonal matrices $[Z^n_{t1}]$ and $[Z^n_{t2}]$ respectively, representing the leakage impedances of the transformers T_1 and T_2.

If the multi-phase transmission line is represented by equivalent or nominal π-circuit parameters, the equivalent single-phase series impedance and the equivalent single-phase shunt admittance can be known with the help of any two equations

among (4.10a)-(4.10d).

Using equations (4.10a) and (4.10b), we get

$$Z^1_{P,eq} = \frac{3}{n}\ \frac{\alpha_{3t1}}{\alpha_{nt1}} \cdot \frac{\alpha_{3t2}}{\alpha_{nt2}} [\, Z^1_P + (Z_1+Z_2)(1+\tfrac{1}{2}\, Z^1_P Y^1_{sh}) + Z_1 Z_2 Y^1_{sh}(1+0.25 Z^1_p Y^1_{sh})] \qquad (4.11)$$

and

$$Y^1_{sh,eq} = \frac{1}{Z^1_{P,eq}}\ [Z^1_P Y^1_{sh} + 2Z_1 Y^1_{sh}(1+0.25 Z^1_P Y^1_{sh})\ ] \qquad (4.12)$$

using equations (4.10b) and (4.10c),

$Z^1_{P,eq}$ is found as in equation (4.11)

and

$$Y^1_{sh,eq} = \frac{1}{Z^1_{P,eq}}\ [Z^1_P Y^1_{sh} + (Z_1+Z_2) Y^1_{sh}(1+0.25 Z^1_P Y^1_{sh}) + Z_1 Z_2 (Y^1_{sh})^2 (1+0.25 Z^1_P Y^1_{sh}) + (Z_1+Z_2).5 Z^1_P (Y^1_{sh})^2 (1+0.25 Z^1_P Y^1_{sh})] \quad (4.13)$$

or

$$Y^1_{sh,eq} = \frac{n}{3}\ \frac{\alpha_{nt1}}{\alpha_{3t1}} \cdot \frac{\alpha_{nt2}}{\alpha_{3t2}}\ Y^1_{sh}(1+.25\ Z^1_P\ Y^1_{sh}) \qquad (4.14)$$

using equations (4.10a) and (4.10c),

$Y^1_{sh,eq}$ is found as in equation (4.14)

$$Z^1_{P,eq} = \frac{3}{n} \cdot \frac{\alpha_{3t1}}{\alpha_{nt1}} \cdot \frac{\alpha_{3t2}}{\alpha_{nt2}} [\, Z^1_P + 2Z_1(1+0.25 Z^1_P Y^1_{sh}) - .5 Z_1 Z^1_P Y^1_{sh} (1+0.25 Z^1_P Y^1_{sh})\ ] \qquad (4.15)$$

From equations (4.11) and (4.15) which give the equivalent single-phase series impedance from the two sets of equations, it is observed that the two values are different. But for the practical purposes, where Z_1 and Z_2 do not differ much, the values obtained from the two equations are almost the same, since the third term under bracket in both the equations contribute insignificantly towards the value. It is also observed that the equivalent single phase series impedance $Z^1_{P,eq}$ decreases with the increasing number of phases while the shunt admittance from equations (4.12), (4.13) and (4.14) increases with the increasing phases. The shunt admittance, from equations (4.12) and (4.13), is almost the same because the 3rd and 4th term under the bracket in equation (4.13) adds little to its value.

4.3 PROBLEM FORMULATION

Suppose there are double-circuit three-phase lines in a power system network which is generally true. There is a problem of transmitting more powers to meet the increased demands at the receiving ends and let us assume that there are two alternatives to accomplish this task viz. 1) the transmission voltage of the existing double circuit three-phase line is increased, say by 73 percent; 2) the existing double circuit three-phase line is converted to a single-circuit six-phase line making its phase to ground voltage equal to $\sqrt{3}$ time that of the existing double circuit three-phase line i.e.

maintaining the same line to line voltage. In order to compare the two options from the viewpoint in two cases of power transfer capability, load flow investigation under two conditions are desirable.

Case a : when the slack bus voltage is fixed

The load flow analysis usually requires one slack bus which is a **reference bus.** As the investigation of the load flow **because** of converting a double circuit three-phase line in either of the options can be interpreted as an equivalent to changing parameters of the line from one value to the another and its effect on the performance thereof; so by assuming slack bus, the slack bus complex generation will change correspondingly from one parameter to other parameter . Since load flow is carried out for planning, it necessarily gives the slack bus generation,and also the other performances with the changing parameters.

Case b : when the complex generation at the slack bus in case a is fixed

The load-flow investigation can, however, be viewed in another way. For example, without affecting the generation of any bus connected to the generator, the effect in the steady-state conditions of the power system network for the two options can be investigated.

There are, however, two types of situations where a single-circuit multi-phase line can be considered to replace a double circuit three-phase transmission line.

1) If the power transmission capability of the existing double-circuit three-phase transmission line is required to be increased to meet the growing demand. This can be accomplished by converting the double-circuit three-phase line to a single-circuit six-phase line with its phase to ground voltage equal to $\sqrt{3}$ times that of the double circuit three-phase lines i.e. maintaining the same line to line voltages in both the cases.

The investigation will facilitate to examine the performance of multi-phase transmission lines transmitting 73% more power in comparision to a double circuit three-phase lines, and also the performance of the multiphase lines when the same power is transmitted at $\sqrt{3}$ times voltage (phase to ground) of the double circuit three-phase lines. Since the voltage between two adjacent phases of multi-phase line has been considered to be equal to (in case of six-phase line) and less than (in case of phases higher than six) that of a double circuit three-phase line, the adjacent phase to phase insulation requirement will be the same for both systems, but the phase to ground insulation requirement will be more in case of multi-phase systems than that in double circuit three-

phase lines. But the insulation is generally provided conservatively and therefore, the multi-phase uprating can take place without restructuring.

2) If an existing double circuit three-phase line is replaced by a single-circuit multi-phase line for transmitting the same power at the same phase to ground voltage retaining the advantages of the multi-phase transmission line compared to a double circuit three-phase line as pointed out by the investigations 1 . Since the phase to ground voltage is the same in both cases, so the phase to ground insulation requirement is the same whereas the adjacent phase to phase insulation requirements for multi-phase system will be equal to $1/\sqrt{3}$ times in case of six-phase transmission system, and even less, in case of transmission line higher than six, than double circuit three-phase lines. This investigation will examine the performance of a multi-phase transmission line in comparision to a double circuit three-phase line at the same phase to ground voltage of the two transmission systems.

The investigations of the system for the two voltage conditions have been carried out under case a and case b. The load flow analysis pertaining to case a is carried out by different methods, such as Gauss-Siedel, Newton-raphson method, in which one of the buses is assumed as a slack bus

(i.e. reference bus) whose voltage magnitude and angle are known, and the voltages of the other buses as well as the slack bus generation are calculated. But the load flow analysis pertaining to case b is hardly discussed [68]. In case b, the reactive power generation instead of the voltage magnitude is known at the slack bus. The angle of the slack bus voltage is known as in case a, and all the bus voltages are calculated.

4.4 CASE STUDIES

This section investigates the load flow analysis of a completely three-phase systems, a combined three-phase and multi-phase systems to compare their performances under the different cases on the two sample networks.

4.4.1 A Three-bus Single-Machine Sample Network

The load flow analysis for the two cases has been carried out on a three-bus single machine sample network [2] as shown in Fig. 4.1. The necessary data are provided in Appendix A.1. The pu parameters of different types of transmission lines have been calculated and have been given in Table A.2. The p.u line parameter for the different configurations, namely, a double-circuit three-phase line, a single circuit six-phase line; the phase to neutral voltage of the multi-phase line is equal to, or $\sqrt{3}$ times that of the double

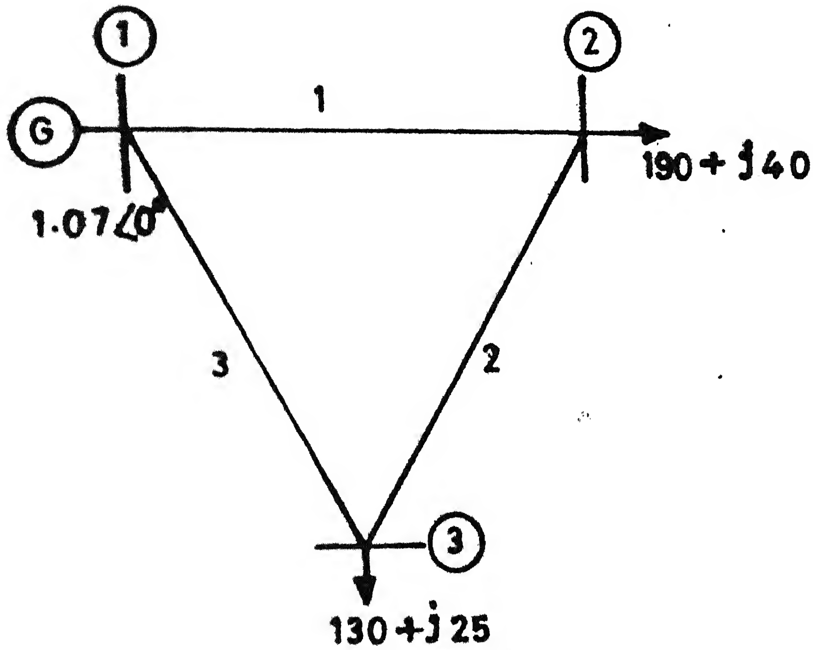


FIG.4.1 A three-bus Single-machine Sample Network.

circuit three-phase lines, and the corresponding parameters are provided in Table 4.1. Two values of each parameters have been given by calculating them with two sets of equations, namely, equations (4.11) and (4.12) and (4.14) and (4.15) and the values are almost identical. The line number between buses 1 and 2 is converted to a single-circuit six-phase and twelve-phase transmission lines for two configurations, namely, the phase to ground voltage of a multi-phase line is $\sqrt{3}$ times that of the double-circuit three-phase line and the phase to ground voltage of the two systems are the same, under cases a and b. In case a, the system has also been investigated with the increased system loading. The leakage reactance of the transformer at each end of the multi-phase line has been taken as 8%.

4.4.2 A Nine Bus Threee-Machine Sample Network

A sample network [55] , shown in Fig. 4.2, has been taken, where line nos. 2,3, and 6 have been assumed as double-circuit three-phase lines, to investigate the impact of conversion of those three-phase lines to multi-phase lines for the purpose of studying the load flow performance of the network. The necessary data of the sample system for a completely three-phase lines have been provided in Appendix B. The pu series impedance and the shunt admittance of the multi-phase line connected to an appropriate transformer at each end of the

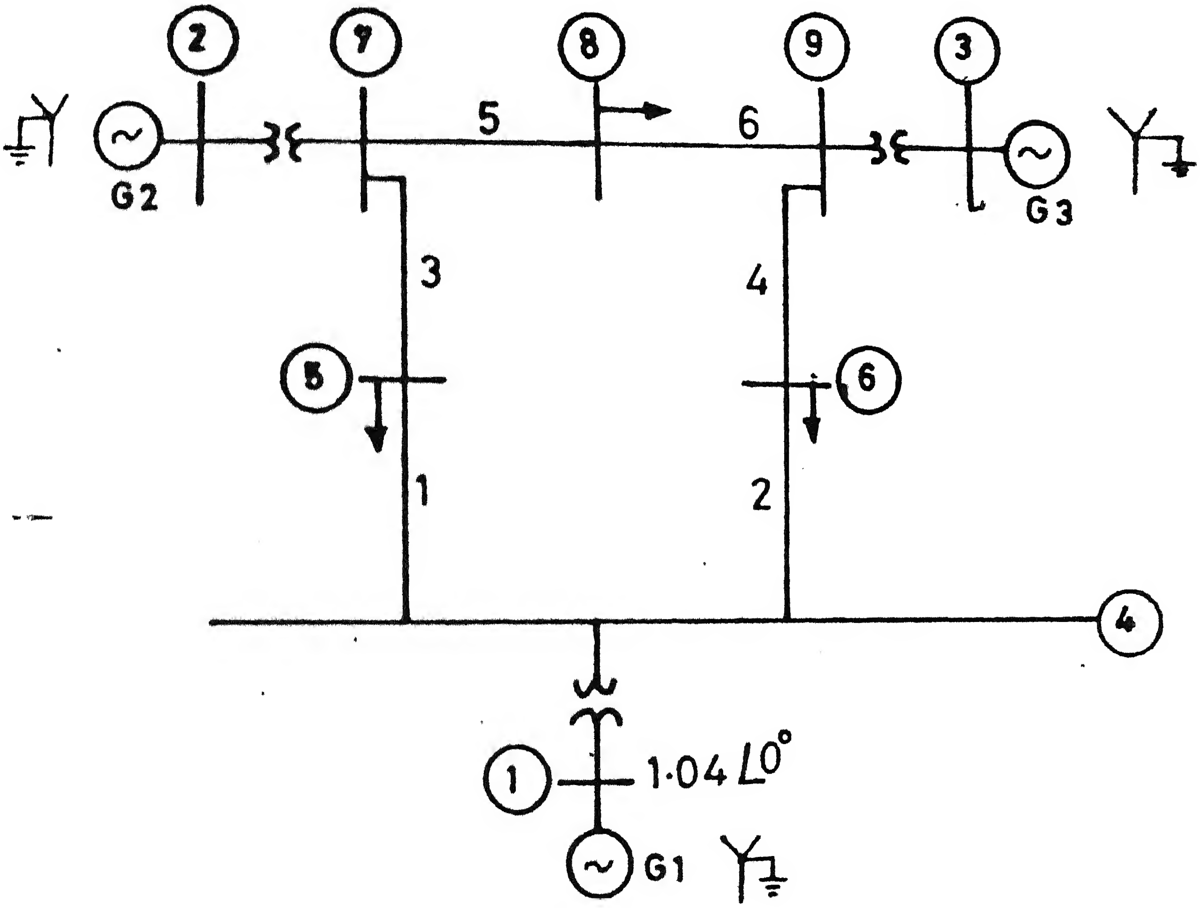


FIG.4.2 A nine-bus three-machine Sample Networ

line have been provided in Table 4.2. Two values of the leakage reactances (i.e., 8% and 5%) of the transformers have been taken to obtain each set of parameters. For, the present load flow study, the p.u. equivalent single-phase series impedance and the shunt impedance corresponding to 8% leakage impedance of the transformer have been used. The load flow investigation has been carried out for case a and case b. Firstly, the line no.2 (double-circuit three-phase line) is converted to a multi-phase line and thereafter line nos. 3 and 6 are also converted to multi-phase lines to investigate the impact of conversions of double-circuit three-phase lines to multi-phase transmission lines.

4.5 RESULTS AND DISCUSSIONS

The load flow study has been carried out on a sample system shown in Fig.4.1where line no.1 is a double-circuit three-phase line, to investigate the impact of conversion from double circuit three-phase line to a single-circuit six-phase or to a twelve-phase line under voltage condition A and B for case a and case b. The bus voltages and the line flows for a six-phase conversion have been provided in Table 4.3. and 4.4. for case a, and those for a twelve-phase conversion in Table 4.5 and 4.6. The bus voltages for case b of a six-phase and a twelve-phase conversions have been provided in Table 4.7, while the line flows and efficiencies in Table 4.8.

TABLE 4.1

Parameters of Multi-Phase Lines for The Sample Network Shown in Fig. 4.1

Type of Line	Voltage * condition	Transformer leakage Impedance .08 Series Imp. Z_p	.08 half shunt adm. $1/2\ Y_{sh}$	.05 Series Imp. Z_p	.05 half shunt adm. $1/2\ Y_{sh}$
6-phase line	A (i)+	.0052+j.059	0.0+j.1642	.0052+j.0491	0.0 +j.164
	(ii)	.0052+j.0592	0.0+j.1634	.0052+j.0492	0.0+j.1634
	B (i)	.0155+j.1770	0.0+j.0547	.0155+j.1473	0.0+j.0547
	(ii)	.0155+j.1776	0.0+j.0545	.0155+j.1476	0.0+j.0545
12-Phase transmission	A (i)	.0026+j.0295	0.0+j.3283	.0026+j.0246	0.0+j.0328
	(ii)	.0026+j.0296	0.0+j.3267	.0026+j.0246	0.0+j0.3267
	B (i)	.0077+j.0885	0.0+j.1094	.0078+j.0737	0.0+j0.1093
	(ii)	.0078+j.0888	0.0+j.1089	.0078+j.0138	0.0+j0.1089

* The configurations in column 2 have the following meanings:

A - Single-circuit multi-phase line with phase to ground voltage $\sqrt{3}$ times that of the double-circuit three-phase line

B - Single-circuit multi-phase line with phase to ground voltage equal to that of the double circuit three-phase line.

+ (i) and (ii) refer to the following meansings:

(i) - Parameters have been obtained using equations (4.11) and (4.12)

(ii) - Parameters have been obtained using equations (4.14) and (4.15).

TABLE 4.2

Parameters of Multi-Phase Lines for the Sample Network Shown in Fig. 4.2

Line num.	Type of line	Voltage Condition	Transformer leakage impedances 0.08: Series Imp. Z_p	0.08: half shunt adm. $1/2\ Y_{sh}$	0.05: series Imp. Z_p	0.05: half shunt adm. $1/2\ Y_{sh}$
2	6-phase line	A (i)	.0056+j.0571	0.0+j.2378	.0056+j.0472	0.0+j.2375
		(ii)	.0057+j.0573	0.0+j0.2361	.0057+j.0473	0.0+j.2361
		B (i)	.0169+j.1712	0.0+j.0793	.0169+j.1415	0.0+j.0792
		(ii)	.017+j.1720	0.0+j.0787	.017+j.142	0.0+j.0787
	12-phase line	A (i)	.0028+j.0285	0.0+j.4755	.0028+j.0236	0.0+j0.4749
		(ii)	.0028+j.0287	0.0+j0.4723	.0028+j.0237	0.0+j0.4723
		B (i)	.0084+j.0856	0.0+j0.1585	.0085+j.0708	0.0+j0.1583
		(ii)	.0085+j.086	0.0+j0.1574	.0085+j.0710	0.0+j0.1574
3	6-phase line	A (i)	.0105+j.0795	0.0+j0.4618	.0106+j.0699	0.0+j.4608
		(ii)	.0107+j0.0803	0.0+j0.4533	.0107+j.0703	0.0+j0.4533
		B (i)	.0316+j0.2385	0.0+j0.1538	.0318+j.2096	0.0+j0.1536
		(ii)	.032+j.2410	0.0+j0.1511	.032+j0.211	0.0+j.1511
	12-phase line	A (i)	.0053+j.0398	0.0+j.9237	.0053+j.0349	0.0+j0.9215
		(ii)	.0053+j.0402	0.0+j0.9067	.0053+j.0352	0.0+j0.9067
		B (i)	.0158+j.1193	0.0+j.3079	.0159+j.1048	0.0+j0.3072
		(ii)	.016+j.1205	0.0+j.3022	.0160+j.1055	0.0+j0.3022
6	6-phase line	A (i)	.0039+j.0599	0.0+j0.3148	.0039+j.05	0.0+j.3143
		(ii)	.0040+j0603	0.0+j0.3118	.0040+j.0503	0.0+j.3118
		B (i)	.0118+j.1796	0.0+j0.1049	.0118+j.1501	0.0+j0.1048
		(ii)	.0119+j.1808	0.0+j0.1039	.0119+j.1508	0.0+j0.1039
	12-phase line	A (i)	.0059+j.0898	0.0+j.2099	.0059+j.0751	0.0+j0.2095
		B (ii)	.0059+j.0904	0.0+j.2079	.0059+j.0754	0.0+j.2079

TABLE 4.3

Bus Voltages of the Sample Network (Fig.4.1)
for Case a (A double Circuit line (no.1) is converted to a single-circuit six-phase line)

Bus No.	Type of Bus	Generation	Load	Voltage magnitude	Angle in degree	Voltage condition
1	Slack	331.43+j113.81	0.0+j0.0	1.07	0.0	
2	P,Q	0.0+j0.0	190+j40	0.9864	-10.25	I
3	P,Q	0.0+j0.0	130+j25	0.9729	-12.5041	
1	Slack	325.79+j68.41	0.0+j0.0	1.07	0.0	
2	P,Q	0.0+j0.0	190+j40.0	1.0329	-6.8129	A
3	P,Q	0.0+j0.0	130+j25	1.0006	-10.5428	
1	Slack	333.44+j162.77	0.0+j0.0	1.07	0.0	
2	P,Q	0.0+j0.0	190+j40	0.9229	-18.393	B
3	P,Q	0.0+j0.0	130+j25	0.9288	-16.7839	
1	Slack	470+j151.97	0.0+j0.0	1.07	0.0	
2	P,Q	0.0+j0.0	330+j70	0.998	-10.9046	II
3	P,Q	0.0+j0.0	130+j25	0.9786	-12.7642	

* In the last column, the voltage condition, refers to the following :

I – a double circuit three-phase line

II – a double circuit three-phase line is converted to a single-circuit multi-phase line under voltage conditions A with increased load.

TABLE 4.4

Line Flows and Transmission Efficiencies of the Sample System shown in Fig.4.1 for Case a

(Line No.1 is converted to a single-circuit 6-Phase line)

Voltage Condition	Line No.	From Bus	To Bus	Line flow P_{ij} (MW)	Q_{ij} MVAR	Efficiency %
I	1	1	2	215.91	72.02	96.68
	2	2	3	18.76	1.46	99.37
	3	1	3	115.52	41.78	96.39
A	1	1	2	226.76	41.62	98.89
	2	2	3	34.26	9.29	98.87
	3	1	3	99.02	26.79	97.06
B	1	1	2	184.25	94.71	96.75
	2	3	2	11.77	1.18	99.57
	3	1	3	149.188	68.06	95.02
II	1	1	2	352.81	112.92	98.17
	2	2	3	16.37	4.59	99.39
	3	1	3	117.98	39.04	96.38

TABLE 4.5

Bus Voltages of the Sample System Shown in Fig. 4.1 for Case a

(Line No.1 is converted to a single circuit 12-phase line)

Bus No.	Type of Bus	Generation	Load	Voltage mag.	Angle in degree	Voltage Condition
1	Slack	324.11+j13.69	0.0+j0.0	1.07	0.0	
2	P,Q	0.0+j0.0	190+j40	1.0573	-3.5699	A
3	P,Q	0.0+j0.0	130+j25	1.0152	-8.7769	
1	Slack	327.55+j97.84	0.0+j0.0	1.07	0.0	
2	P,Q	0.0+j0.0	190+j40.0	1.0073	-9.8558	B
3	P,Q	0.0+j0.0	130+j25.0	0.9846	-12.1965	
1	Slack	466.38+j69.31	0.0+j0.0	1.07	0.0	
2	P,Q	0.0+j0.0	330+j70.0	1.0428	-5.5717	II
3	P,Q	0.0+j0.0	130+j25	1.0066	-9.8657	

TABLE 4.6

Line Flows and Transmission Efficiencies of the Sample Shown in Fig.4.1 for Case a

(Line No.1 is converted to a Single-circuit 12-phase line)

Line No.	From Bus	To Bus	Syst. configuration A			Syst. confi. B			Syst. confi. II		
			P_{ij} (MW)	Q_{ij} (MVAR)	Effi. %	P_{ij} (MW)	Q_{ij} (MVAR)	Effi. %	P_{ij} (MW)	Q_{ij} (MVAR)	Effi. %
1	1	2	240.8	-5.31	99.44	214.32	62.16	98.36	373.32	45.77	99.11
2	2	3	49.46	13.35	98.47	20.8	5.73	99.26	39.99	10.68	98.72
3	1	3	83.3	19.0	97.58	113.23	35.67	96.56	93.06	23.53	97.26

TABLE 4.7

Bus Voltages of the Sample System (Fig. 4.1) for Case b

Bus No.	Generation	Load	Voltage	Angle in degree	Voltage Condition	Type* of Multi-Phase Line
1	331.39+j113.32	0.0+j0.0	.8935	0.0		
2	0.0+j0.0	190+j40	.8362	-10.14	A	
3	0.0+j0.0	130+j25	0.7910	-16.075		six-phase line
1	331.39+j113.32	0.0+j0.0	1.2415	0.0		
2	0.0+j0.0	190+j40	1.1401	-12.81	B	
3	0.0+j0.0	130+j25	1.1434	-11.75		
1	331.39+j113.32	0.0+j0.0	.7396	0.0		
2	0.0+j0.0	190+j40	.7028	-7.7814	A	
3	0.0+j0.0	130+j25	.6156	-21.0299		12-Phase line
1	331.39+j113.32	0.0+j0.0	1.0103	0.0		
2	0.0+j0.0	190+j40	0.9398	-11.27	B	
3	0.0+J0.0	130+j25	0.9129	-14.02		

* Type of multi-phase line refers to a multi-phase transmission line in which a double circuit three-phase line (No.1) has been converted.

TABLE 4.8

Line flows and Transmission Efficiencies of the Sample Network (Fig. 4.1) for Case b

Line No.	From Bus	To Bus	Line flows P (MW)	Line flows Q (MVAR)	Effici. η %	Voltage condition	Type of multi-phase line
1	1	2	229.83	73.0	98.29		
2	2	3	35.198	12.976	98.16	A	
3	1	3	100.84	40.37	95.37		six-phase line
1	1	2	183.23	66.29	97.85		
2	3	2	11.57	3.196	99.72	B	
3	1	3	147.32	47.1	96.67		
1	1	2	245.386	68.69	98.68		
2	2	3	51.97	26.09	95.82	A	
3	1	3	85.79	44.64	93.71		12-Phase line
1	1	2	216.094	72.44	88.1		
2	2	3	21.15	6.51	99.1	B	
3	1	3	114.46	40.95	96.0		

From Table 4.3, it is observed that, when the slack bus voltage magnitude is fixed at 1.07 p.u., the reactive power at the slack bus are different for the different configurations. For example, the required generation for an original (i.e. completely) three-phase system at the slack bus is 113.81 MVAR while it reduces to 68.41 MVAR for voltage condition A and increases to 162.77 for voltage condition B and 151.97 for configuration II. It is also observed that the voltage and angle improves under configuration A and they deteriorates under configuration B in comparision to configuration I, while in configuration II the voltages and angles are comparable to those under configuration I. When line No.1 is converted to a twelve-phase line, it is observed from Table 4.5 that in all configurations, the voltage and angle improves in comparision to those in configuration I, bu t the voltage and angle in condition A & II have improved more than those of condition B, a similar observation as from Table 4.3, and also, the reactive power required under configuration A & II are less than those under configuration B ; however, for this twelve-phase conversion, the reactive power in all configurations are less than that of configuration I.

From Table 4.7, where, the load flow study has been carried out by assuming the reactive power (though normally not true) of the slack bus (generation) fixed at 113.32 MVAR,

it is observed that the bus voltages under voltage condition B for six-phase conversion are higher while those under voltage condition A are lower than the voltages under configuration I. For twelve-phase conversion, the voltages are even lower than six-phase configuration under voltage condition A, and higher than those for six-phase conversion under voltage condition B but it is still lower than those in configuration I.

Further, the load flow investigation has been carried out on a sample system network shown in Fig. 4.2 where three lines, namely line nos. 2,3 and 6 have been considered as double-circuit three-phase lines, to investigate the system under different configuration especially with a view to examine the impact of increasing number of conversion of double-circuit three-phase lines to multi-phase lines for various cases.

The bus voltages and the line flows of the sample network under configuration I have been provided in Tables 4.9 and 4.10. The slack bus voltage has been assumed as 1.04 for case a. The slack bus generation for six-phase and twelve-phase conversions under different voltage conditions and system configurations have been provided in Table 4.11. The bus voltages and the line flows for six-phase conversion have been provided in Tables 4.12 and 4.13 for case a, while those for twelve-phase conversion in Tables 4.14 and 4.15 respectively.

The bus voltages and the line flows for the sample network (Fig.4.2) have been provided for six-phase conversion in Tables 4.16 and 4.17 respectively for case b where the reactive power of the slack bus has been fixed at 27.03 MVAR, and those for twelve-phase conversion in Tables 4.18 and 4.19 respectively.

It is observed from Tables 4.9 and 4.11 that the slack bus reactive power are different for different voltage conditions, namely voltage conditions A and B, and different system configurations i.e., configuration I, configuration II and configuration III for six-phase and twelve-phase conversions. For example, the slack bus reactive power is 27.03 MVAR for a completely three-phase system, 25.44 MVAR for system configuration I, 148.22 for configuration II and 246.28 MVAR for configuration III for six-phase conversion under voltage condition A, and the corresponding values for different system configurations under voltage condition B are 30.82 MVAR, 45.02 MVAR and 47.3 MVAR. For twelve-phase conversion, the respective values are 99.55 MVAR, 444.92 MVAR and 805.52 MVAR under voltage condition A and .15 MVAR, 60.84 MVAR and 107.69 under voltage condition B. It is observed that the required reactive power for twelve-phase conversion is higher than those for six-phase conversion. Moreover, the active powers have also changed substantially with respect to that of the

TABLE 4.9

Bus Voltages and Generations of the Sample System shown in Fig.4.2

Bus No.	Type of Bus	Generation	Load	Volt. mag.	Angle in degree
1	Slack	71.64+j27.03	0.0+j0.0	1.04	0.0
2	P, V	163.0+j6.7	0.0+j0.0	1.025	9.27
3	P, V	85.0+j10.9	0.0+j0.0	1.025	4.66
4	P,Q	0.0+j0.0	0.0+j0.0	1.0258	-2.22
5	P,Q	0.0+j0.0	125+j50.0	.9957	-3.98
6	P,Q	0.0+j0.0	90+j30	1.0127	-3.687
7	P,Q	0.0+j0.0	0.0+j0.0	1.0258	3.719
8	P,Q	0.0+j0.0	100+j35	1.0159	0.7273
9	P,Q	0.0+j0.0	0.0+j0.0	1.0323	1.96

TABLE 4.10

Line Flows and Trans. Efficiencies of a Sample System shown in Fig. 4.2 for a Completely Three-Phase Lines.

Line No.	From Bus	To Bus	Line P_{ij} (MW)	flows Q_{ij} (MVAR)	Efficiency %
1	4	5	40.928	22.875	99.37
2	4	6	30.698	1.0328	99.458
3	7	5	86.61	8.365	97.35
4	9	6	60.808	18.078	97.78
5	7	8	76.37	.7737	99.377
6	9	8	24.18	3.084	99.63

TABLE 4.11

Slack Bus Generations for Different Configurations of the Sample System Shown in Fig.4.2 for Case a

Type of Line	Voltage Condition	Slack bus Generation for different configurations		
		config. I	config. II	config. III
6-phase line	A	71.14-j25.44	69.85-j148.22	72.57-j246.28
	B	71.72+j30.82	72.12+j45.02	72.21+j47.3
12-phase Line	A	70.82-j99.55	76.32-j444.92	107.19-j805.52
	B	71.32+j.15	69.88-j60.84	70.32-j107.69

Configuration I - when line no.2 is converted to a single-circuit multi-phase transmission line

Configuration II- when line nos. 2 and 3 are converted to single circuit multi-phase transmission lines

Configuration III - when line nos. 2,3 and 6 are converted to single circuit multi-phase transmission line.

TABLE 4.12

Bus Voltages of the Sample System Shown in Fig. 4.2 for Case a

(conversion of double-circuit 3-phase lines to single circuit six-phase lines)

Voltage condition	Bus No.	Voltage magnitude and Angle of the buses					
		config. I		config. II		config. III	
		Voltage	Angle	Voltage	Angle	Voltage	Angle
A	1	1.04	0.0	1.04	0.0	1.04	0.0
	2	1.074	8.38	1.22	3.67	1.3466	2.2267
	3	1.077	4.28	1.199	1.55	1.3445	-0.1321
	4	1.0548	-2.14	1.1228	-1.97	1.1771	-1.95
	5	1.0337	-3.775	1.174	-3.238	1.2698	-3.3148
	6	1.0611	-3.11	1.146	-3.22	1.2265	-3.2484
	7	1.074	3.32	1.2196	-0.253	1.3456	-0.9964
	8	1.067	0.64	1.2060	-1.95	1.354	-2.45
	9	1.084	1.84	1.2055	-0.42	1.3498	-1.7047
B	1	1.04	0.0	1.04	0.0	1.04	0.0
	2	1.0174	9.02	.9928	12.06	.9836	12.07
	3	1.015	4.055	0.9917	6.01	.9924	6.25
	4	1.0237	-2.22	1.0159	-2.25	1.0146	-2.253
	5	.9917	-4.15	0.9774	-4.565	.9739	-4.63
	6	1.0016	-4.77	.9833	-4.075	.9827	-3.95
	7	1.0183	3.38	.9938	6.134	.9848	6.03
	8	1.007	0.218	.9823	2.47	.972	2.24
	9	1.0225	1.304	.9994	3.13	1.0	3.377

TABLE 4.13

Line Flows and Trans. Efficiencies of Sample System Shown in Fig. 4.2 for Case a (Convesion of double circuit 3-phase lines to single circuit six-Phase lines)

Volt. cond.	Line No.	Line flows and efficiencies								
		config. I			config. II			config. III		
		P (MW)	Q (MVAR)	η %	P (MW)	Q (MVAR)	η %	P (MW)	Q (MVAR)	η %
	1(4-5)	39.2	-12.27	99.5	25.85	82.12	98.24	26.26	143.23	95.09
	2(4-6)	31.94	40.75	99.8	43.99	80.39	99.35	46.3	138.15	98.82
	3(7-5)	88.2	2.77	97.49	100.55	9.96	99.03	101.42	33.08	98.6
A	4(9-6)	59.29	17.55	98.02	47.08	-5.86	98.16	46.34	55.04	95.5
	5(7-8	74.79	4.94	99.44	62.44	5.49	99.62	61.58	35.55	99.67
	6(9-8)	25.7	2.94	99.65	37.92	19.76	99.68	38.65	68.32	99.9
	1	44.11	24.78	99.32	51.92	31.75	99.18	53.36	34.21	99.14
	2	27.6	2.78	99.48	20.19	9.41	99.41	18.85	9.1	99.43
	3	83.33	10.22	97.4	75.33	11.12	97.56	73.89	13.13	97.56
B	4	64.06	17.46	97.57	71.94	20.32	97.2	73.35	19.75	97.14
	5	79.66	-.858	99.33	87.65	-.95	99.23	89.09	2.65	99.2
	6	20.93	2.38	99.6	13.05	5.04	99.63	11.64	4.49	99.64

TABLE 4.14

Bus Voltages of Sample System Shown in Fig. 4.2 for Case a (conversion of double circuit 3-phase lines to single circuit 12-phase lines)

Volt. Cond.	Bus No.	Voltage magnitude and Angle of the bus voltages: config.I mag.	config.I Angle	config.II mag.	config.II Angle	config. III mag.	config. III Angle
A	1	1.04	0.0	1.04	0.0	1.04	0.0
	2	1.1301	7.52	1.5252	0.0933	1.9264	-2.464
	3	1.1323	3.95	1.4526	-0.505	1.9397	-3.87
	4	1.0958	-2.051	1.2871	-1.882	1.4873	-2.28
	5	1.0825	-3.527	1.4754	-3.307	1.8238	-4.29
	6	1.109	-2.573	1.3247	-2.57	1.5714	-3.04
	7	1.13	2.95	1.524	-2.42	1.925	-4.03
	8	1.124	.5802	1.494	-3.27	1.962	-4.84
	9	1.1388	1.7344	1.4574	-1.85	1.943	-4.626
B	1	1.04	0.0	1.04	0.0	1.04	0.0
	2	1.052	8.67	1.1288	5.98	1.197	4.83
	3	1.054	4.29	1.1201	2.72	1.1948	1.732
	4	1.04	-2.17	1.0744	-2.06	1.1	-2.03
	5	1.0159	-3.9	1.0856	-3.57	1.1314	-3.57
	6	1.04	-3.58	1.0874	-3.66	1.1317	-3.62
	7	1.0525	3.4	1.1287	1.4	1.1966	0.753
	8	1.0413	0.56	1.118	-0.87	1.1963	-1.31
	9	1.0612	1.736	1.1267	0.4578	1.2	-0.2575

TABLE 4.15

Line Flows and Trans. Efficiencies of Sample System (Fig. 4.2) for Case a (conversion of double circuit 3-phase lines to single Circuit 12-Phase lines)

Volt. cond.	Line No.	Line flows and efficiency								
		config. I			config. II			config. III		
		P (MW)	Q (MVAR)	η %	Q (MW)	Q (MVAR)	η %	P (MW)	Q (MVAR)	η %
A	1	37.49	-2.64	99.65	21.79	301.6	77.05	42.36	611.18	62.44
	2	33.32	110.14	99.72	54.52	251.82	98.97	64.83	546.0	95.1
	3	89.76	.808	97.63	109.13	40.48	99.15	101.87	135.34	96.74
	4	57.79	-14.46	98.2	38.32	67.0	94.0	45.69	347.44	60.59
	5	73.24	-7.13	99.51	53.96	40.03	99.57	61.12	133.12	99.44
	6	27.2	.205	99.7	46.67	79.94	99.34	39.3	359.487	99.78
B	1	40.61	16.56	99.47	33.98	27.91	99.62	34.37	54.36	99.25
	2	30.7	19.12	99.75	35.9	37.496	99.66	35.94	62.13	99.38
	3	86.82	4.92	97.44	92.34	6.624	98.7	92.17	12.42	98.6
	4	60.64	18.16	97.9	55.23	7.543	98.17	15.48	11.71	97.8
	5	76.17	3.409	99.41	70.65	.27	99.51	70.82	17.336	99.57
	6	24.35	3.39	99.65	29.77	6.787	99.7	29.51	25.62	99.87

TABLE 4.16

Bus Voltages of the Sample System Shown in Fig. 4.2 for case b

(conversion of double-circuit 3-phase lines to single circuit six-phase lines)

Volt. cond.	Bus No.	config.I magnitude	config.I Angle	config II magnitude	config II Angle	config. III magnitude	config. III Angle
	1	.9775	0.0	.8651	0.0	.81	0.0
	2	.9399	11.57	.8227	10.66	.7719	12.43
	3	.9481	6.126	.8166	5.83	.7742	5.61
	4	.9625	-2.51	.8484	-3.22	.7924	-3.68
A	5	.9196	-4.42	.8191	-4.815	.762	-5.32
	6	.9553	-3.625	.8328	-5.25	.7778	-6.16
	7	.9418	4.96	.8269	2.056	.7778	2.663
	8	.9326	1.4659	.807	-1.645	.7713	-1.39
	9	.9563	2.97	.8267	1.6	.785	0.917
	1	1.0444	0.0	1.0575	0.0	1.0592	0.0
	2	1.0278	8.79	1.0417	10.75	1.039	10.6
	3	1.0254	3.92	1.0402	5.26	1.0459	5.4
	4	1.0303	-2.197	1.0435	-2.143	1.0452	-2.1355
B	5	1.0001	-4.102	1.0131	-4.354	1.0138	-4.39
	6	1.0107	-4.7166	1.0243	-3.88	1.028	-3.75
	7	1.0285	3.2685	1.0423	5.37	1.0396	5.196
	8	1.0177	.1652	1.0321	2.042	1.0284	1.794
	9	1.0328	1.2286	1.0474	2.64	1.0531	2.81

TABLE 4.17

Line Flows and Trans. Efficiencies of Sample System Shown in Fig.4.2 for Case b (conversion of double circuit 3-phase lines to single circuit 6-phase lines)

Volt. cond.	Line No.	Lines flows and efficiency								
		config. I			config. II			config. III		
		P (MW)	Q (MVAR)	η %n	P (MW)	Q (MVAR)	η %	P (MW)	Q (MVAR)	η %
	1	39.87	-36.27	99.03	25.87	20.25	99.26	23.42	20.403	99.17
	2	31.94	12.76	99.78	45.64	2.3	99.57	48.11	1.497	99.513
	3	88.22	10.94	96.81	101.0	30.28	98.42	103.75	18.73	98.15
A	4	59.57	25.92	97.4	45.9	22.85	97.2	43.38	14.88	97.23
	5	74.97	1.22	99.27	61.9	12.44	99.17	59.16	2.428	99.16
	6	25.61	-10.218	99.47	38.98	5.50	99.24	41.52	3.17	99.69
	1	44.06	22.71	99.36	51.77	22.578	99.34	53.11	23.74	99.32
	2	27.55	1.22	99.5	19.84	1.434	99.61	18.51	.276	99.64
	3	83.32	9.34	97.46	75.24	7.48	97.75	73.86	8.92	97.8
B	4	64.068	17.069	97.65	72.07	17.85	97.43	73.44	17.026	97.41
	5	79.66	.284	99.35	87.73	1.14	99.31	89.11	.21	99.29
	6	20.91	2.08	99.67	12.904	2.976	99.69	11.54	2.195	29.7

TABLE 4.18

Bus Voltages of the Sample System (Fig. 4.2) for Case b

(conversion of double circuit 3-phase lines to single circuit 12-phase lines).

Volt. cond.	Bus No.	Magnitudes and Angles of Bus Voltages					
		config. I		config. II		config. III	
		Magnitude	Angle	Magnitude	Angle	Magnitude	Angle
	1	.9162	0.0	.7391	0.0	.6705	0.0
	2	.8444	15.02	.6763	12.56	.6061	16.9
	3	.864	8.22	.6609	8.257	.6153	6.43
	4	.9004	-2.8786	.7202	-4.44	.6505	-5.42
A	5	.8395	-4.95	.6877	-5.62	.6152	-5.95
	6	.8969	-3.5318	.7088	-6.19	.6404	-7.886
	7	.8481	6.83	.6868	-0.1064	.6223	1.244
	8	.84	2.57	.6537	-4.53	.6223	-3.38
	9	.8733	4.425	.6748	1.845	.6319	-0.908
	1	1.0065	0.0	.9409	0.0	.9021	0.0
	2	.9801	10.32	.9083	10.32	.873	11.16
	3	.984	5.237	.9051	5.24	.8752	5.34
	4	.992	-2.37	.9254	-2.71	.886	-2.95
B	5	.9554	-4.24	.8962	-4.54	.8558	-4.95
	6	.9814	-3.88	.9077	-4.79	.8594	-5.25
	7	.9814	4.238	.9107	3.25	.876	3.5
	8	.9718	.9716	.8956	-0.2385	.8674	-0.28
	9	.9918	2.31	.914	1.79	.8843	1.71

TABLE 4.19

Line Flows and Trans. Efficiencies of Sample System Shown in Fig.4.2 for Case b

(conversion of double circuit 3-phase lines to Single Circuit 12-phase lines)

Volt. Cond.	Line No.	Line flows and efficiencies								
		config. I			config. II			config. III		
		P (MW)	Q (MVAR)	η %	P (MW)	Q (MVAR)	η %	P (MW)	Q (MVAR)	η %
A	1	39.35	53.28	98.37	15.04	21.32	98.25	7.44	22.21	97.68
	2	32.85	30.32	99.87	56.55	0.47	99.63	64.0	2.67	99.54
	3	89.57	14.95	96.01	111.56	53.53	98.73	119.8	32.32	98.36
	4	58.9	34.84	96.6	35.029	26.91	96.15	27.16	14.77	97.14
	5	73.96	1.76	99.13	51.39	23.87	98.81	43.03	6.202	99.05
	6	26.63	18.133	99.18	49.92	6.16	98.63	57.65	7.42	99.68
B	1	40.92	29.74	99.22	34.46	20.67	99.33	34.36	21.1	99.27
	2	30.79	6.05	99.71	37.06	2.56	99.56	37.21	1.802	99.54
	3	86.86	9.757	97.1	92.48	20.18	98.2	92.74	14.0	98.04
	4	60.715	22.14	97.58	54.57	20.94	97.43	54.48	15.39	97.28
	5	76.21	.885	99.32	70.42	6.72	99.25	70.22	1.128	99.22
	6	24.36	6.79	99.56	30.32	4.79	99.48	30.46	1.12	99.72

completely three-phase system. It means that slack bus generator should generate required values of reactive powers if possible or the generator should be replaced by another one if necessary to generate required power to keep the slack bus voltage constant for all conditions. The original generator if employed will limit the number of conversions, however the reactive power of the generator will have to be adjusted.

The comparision of Tables 4.9, 4.12 and 4.13 reveals that the bus voltages increase under voltage condition A and they decrease under voltage condition B for six-phase uprating while they increase under both conditions for twelve-phase, with the increasing number of conversions of double-circuit three-phase lines to multi-phase lines for case a.

When Table 4.9 is compared to the results of the load-flow study for case b in Tables 4.16 and 4.18, it is observed that the bus voltages decrease under voltage condition A for six-phase conversion and under voltage conditions A & B for twelve-phase conversion for the three types of system configurations. But under voltage condition B, the bus voltages increase for configuration I and II while for system configuration III, som e bus voltages have decreased in comparision to those for case Ia; however, they are higher than those of the original three-phase systems.

From Tables 4.10 and 4.13, the power flowing through lines 2 and 3 are (40.9+j22.8) and (30.69+j1.0328) in the original completely three-phase system while they are (39.2+j 12.27), (31.94+j40.75) for configuration I, (25.85+j82.12), (43.99+j80.4) for configuration II and (26.26+j143.23), (46.3+j138.15) for configuration III, and therefore, it may be likely that original double circuit three-phase line may not be able for the required conversion for case a under voltage condition A. But the line flows in lines 2 and 3 have not changed much.

If the two options for increasing the power transmission capability of a double-circuit three-phase line as mentioned in section 4.3 is considered, it is observed from equations (4.11) and (4.12), or (4.14) and (4.15) the equivalent single-phase parameters for double circuit three-phase line with increased transmission voltage will be the same as those of a single-circuit six-phase line if the leakage impedances of the transformers in each option is assumed to be the same; since the transmission of more power by increasing the voltage of a double-circuit three-phase line will also require a three-phase/three-phase transformer at the each end of the three-phase line similar to the uprating of a double-circuit three-phase line to a single-circuit six-phase line.

4.6 CONCLUSION

The equivalent single-phase parameters for π-circuit representation of a multi-phase line integrated with a three-phase wye/n-phase star connected transformer at each end of the line under different phase to ground voltages have been obtained to analyse the balanced network on a single-phase basis. The impact of converting double-circuit three-phase to single-circuit multi-phase lines has been studied. It is observed from the results of the load flow investigation that,

1. For the same slack bus reactive power, the bus voltages increase when the phase to ground voltage of the single-circuit six-phase line is the same as that of the double-circuit three-phase line while the bus voltages decrease when the phase to ground voltage of the line is $\sqrt{3}$ times that of the three-phase line, and

2. For the same slack bus voltage, the bus voltages decrease when the phase to ground voltage of the multi-phase line is the same as that of the double-circuit three-phase line while they increase when the phase to ground voltage is increased to $\sqrt{3}$ times that of the three-phase line.

CHAPTER 5

TRANSIENT STABILITY ANALYSIS OF SIX-PHASE SYSTEMS

5.1 INTRODUCTION

In the previous chapter, the load flow investigation has been carried out to study the comparative performance of different configurations of three-phase and six-phase transmission systems. This chapter presents the transient stability investigation of the mixed three-phase and six-phase as well as completely six-phase systems in order to compare their performances with that of three-phase systems. Since, in the transient stability analysis, the model of a synchronous machine is required and the various models of three-phase machines, depending upon the accuracy desired in the analysis are available in the literature [55,69].

The section 5.2 starts with the detailed mathematical representation of six-phase synchronous machine; section 5.2.1 presents the direct and quadrature - axis transient and subtransient inductances, while section 5.2.2 deals with the different time constants for a six-phase synchronous machine. In section 5.3, a case study has been taken for transient stability investigation based upon the sample network shown in Fig.5.1. The system representation for this study has been discussed in section 5.3.2. In section 5.3.3, the

different system configurations of the sample network has been presented. The results obtained from the investigation have been presented and discussed in section 5.3.4. Finally, the chapter concludes in section 5.4.

5.2 MATHEMATICAL REPRESENTATION OF MULTI-PHASE (SIX-PHASE) SYNCHRONOUS MACHINE

The synchronous machine is represented by the different types of model, depending upon the desired accuracy and the requirement of the analysis. These models require different types of inductances, such as direct- and quadrature-axis transient and subtransient inductances, and also various types of time constants of the machine. The inductances and the time constants of a multi-phase (six-phase) machine have been calculated in the following subsections.

5.2.1 Direct and Quadrature Axis Transient and Subtransient Inductances

These inductances of a multi-phase (six- phase) synchronous machine have been derived with the help of the flux linkages and voltage equations in d q o component in equations (2.38) and (2.40).

Since transformer voltages $P\Psi_d$ and $P\Psi_q$ are much smaller than the speed $\omega\Psi_d$ and $\omega\Psi_q$, and therefore, neglecting these voltages, the equations (2.38) and (2.40) in the

matrix form can be written as follows.

$$\Psi_d = L_d i_d + \sqrt{3}\ L_{mF} i_F + \sqrt{3}\ L_{mD} i_D \tag{5.1}$$

$$\Psi_q = L_q i_q - \sqrt{3}\ L_{mG} i_G - \sqrt{3}\ L_{mQ} i_Q \tag{5.2}$$

$$\Psi_F = \sqrt{3}\ L_{mF} i_d + L_{FF} i_F + L_{FD} i_D \tag{5.3}$$

$$\Psi_G = -\sqrt{3} L_{mG} i_q + L_{GG} i_G - L_{GQ} i_Q \tag{5.4}$$

$$\Psi_D = \sqrt{3}\ L_{mD}\ i_d + L_{FD} i_F + L_{DD} i_D \tag{5.5}$$

$$\Psi_Q = -\sqrt{3}\ L_{mQ} i_q - L_{GQ} i_G + L_{QQ} i_Q \tag{5.6}$$

and

$$v_d = -r_a i_d - \omega \Psi_q \tag{5.7}$$

$$v_q = -r_a i_q + \omega \Psi_d \tag{5.8}$$

$$v_F = -r_F i_F - P\Psi_F \tag{5.9}$$

$$V_G = 0 = -r_G i_G - P\Psi_G \tag{5.10}$$

$$V_D = 0 = -r_D i_D - P\Psi_D \tag{5.11}$$

$$V_Q = 0 = -r_Q i_Q - P\Psi_Q \tag{5.12}$$

From equations (5.1) and (5.8), we get,

$$v_q = -r_a i_q + \omega L_d i_d + \sqrt{3}\ \omega\ L_{mF} i_F + \sqrt{3}\omega\ L_{mD} i_D \tag{5.13}$$

From equations (5.3) and (5.5), we have,

$$\begin{bmatrix} i_F \\ \\ i_D \end{bmatrix} = \frac{1}{\Delta 1} \begin{bmatrix} \Psi_F L_{DD} - \Psi_D L_{FD} + (\sqrt{3}\ L_{mD} L_{FD} - \sqrt{3}\ L_{DD} L_{mF}) i_d \\ \\ -\Psi_F L_{FD} + \Psi_D L_{FF} + (\sqrt{3}\ L_{mF} L_{FD} - \sqrt{3}\ L_{FF} L_{mD}) i_d \end{bmatrix} \tag{5.14}$$

where $\Delta 1 = (L_{FF}L_{DD}-L_{FD}^2)$

Substituting i_F and i_D from equation (5.14) in (5.13), the following equation is obtained.

$$v_q = - r_a i_q + \omega \left[L_d - \frac{3}{\Delta 1}(L_{FF}L_{mD}^2 + L_{mF}^2 L_{DD} - 2L_{mF}L_{mD}L_{FD}) \right] i_d$$
$$+ \frac{\sqrt{3}\,\omega\Psi_F}{\Delta 1}(L_{mF}L_{DD}-L_{mD}L_{FD}) + \frac{\sqrt{3}\,\omega\Psi_D}{\Delta 1}(L_{mD}L_{FF}-L_{mF}L_{FD}) \quad (5.15)$$

From equations (5.2) and (5.7), we have,

$$v_d = - r_a i_d - \omega (L_q i_q - \sqrt{3}\, L_{mG} i_G - \sqrt{3}\, L_{mQ} i_Q) \quad (5.16)$$

From equations (5.2) and (5.4)

$$\begin{bmatrix} i_G \\ \\ i_Q \end{bmatrix} = \frac{1}{\Delta 2} \begin{bmatrix} \Psi_G L_{QQ} + \Psi_Q L_{GQ} + (\sqrt{3} L_{QQ} L_{mG} + \sqrt{3}\, L_{mQ} L_{GQ}) i_q \\ \\ \Psi_G L_{GQ} + \Psi_Q L_{GG} + (\sqrt{3} L_{GQ} L_{mG} + \sqrt{3}\, L_{mQ} L_{GG}) i_q \end{bmatrix} \quad (5.17)$$

where $\Delta 2 = (L_{GG}L_{QQ}-L_{GQ}^2)$

Substituting i_G and i_Q from equation (5.17) in equation (5.16), the following expression is obtained.

$$v_d = - r_a i_d - \omega \left[L_q - \frac{3}{\Delta 2}(L_{QQ}L_{mG}^2 + L_{mQ}^2 L_{GG} + 2L_{mG}L_{GQ}L_{mQ}) \right] i_q$$
$$+ \frac{\sqrt{3}\,\omega\Psi_G}{\Delta 2}(L_{mG}L_{QQ}+L_{mQ}L_{GQ}) + \frac{\sqrt{3}\omega\Psi_Q}{\Delta 2}(L_{mG}L_{GQ}+L_{mQ}L_{GG}) \quad (5.18)$$

From equations (5.15) and (5.18), the subtransient inductances are

$$L_d'' = L_d - \frac{3}{\Delta 1}(L_{FF}L_{mF}^2 + L_{mF}^2 L_{DD} - 2L_{mF}L_{mD}L_{FD}) \qquad (5.19)$$

and

$$L_q'' = L_q - \frac{3}{\Delta 2}(L_{QQ}L_{mG}^2 + L_{mQ}^2 L_{GG} + 2L_{mG}L_{QQ}L_{mQ}) \qquad (5.20)$$

Further, when the damper windings D and Q are not considered, we approach the transient state, and hence transient inducances are

$$L_d' = L_d - 3\; L_{mF}^2 / L_{FF} \qquad (5.21)$$

and $$L_q' = L_q - 3\; L_{mG}^2 / L_{GG} \qquad (5.22)$$

where d and q refer to direct and quadrature axis quantities respectively.

The expressions (5.21) and (5.22) give the direct and quadrature axis transient inductance of the machine; while expressions (5.19) and (5.20), the direct and quadrature axis subtransient inductances of the six-phase machine.

5.2.2 Different Time Constants of the Six-Phase Machine

The different time constants of the six-phase synchronous machine have been obtained in this section. These time constants are required for the different types of synchronous machine model.

Direct-axis transient open-circuit time constat T'_{do}:

In this case, a sudden voltage is applied to the field winding keeping armature open circuited without damper winding.

From equations (5.3) and (5.9), the field voltage equation become

$$L_{FF}Pi_F + r_F i_F = -v_F \tag{5.23}$$

Thus, $$T'_{do} = L_{FF}/r_F \tag{5.24}$$

Direct-axis transient short-circuit time constant T'_d :

In this case the condition is similar to above except that the stator windings are short-circuited.

From equations (5.3) and (5.9),

$$\sqrt{3}\ L_{mF}Pi_d + L_{FF}Pi_F + r_F i_F = -v_F \tag{5.25}$$

From equations (5.8) and (5.1), we have,

$$L_d i_d + \sqrt{3}\ L_{mF}\ i_F = 0 \tag{5.26}$$

Substituting values of Pi_d from (5.26) in (5.25),

$$[\ L_{FF} - (\sqrt{3}\ L_{mF})^2/L_d]\ Pi_F + r_F i_F = -v_F \tag{5.27}$$

From equation (5.27),

$$T'_d = \frac{L_{FF}[\ L_d - (\sqrt{3}L_{mF})^2/L_{FF}\ ]}{L_d r_F} = \frac{L'_d}{L_d}\ T'_{do} \tag{5.28}$$

Similarly, the quadrature-axis open circuit transient time constants are given by

$$T'_{qo} = \frac{L_{GG}}{r_G} \; ; \quad T'_q = \frac{L'_q}{L_q} T'_{qo} \tag{5.29}$$

Direct Axis Subtransient open-circuit time constant T''_{do}:

The condition is similar to above except for the presence of damper windings.

From equations (5.3) and (5.9), we have,

$$L_{FF}Pi_F + L_{FD}Pi_D + r_Fi_F = -v_F \tag{5.30}$$

Similarly, from equations (5.5) and (5.11)

$$L_{FD}Pi_F + L_{DD}Pi_D + r_Di_D = 0$$

At $t = 0^+$, $\Psi_D = 0$, hence from equation (5.5),

$$L_{FD}i_F + L_{DD}i_D = 0 \tag{5.32}$$

Substituting i_F from equation (5.32) in equation (5.30), and Pi_F from equation (5.30) in (5.31), the following differential equation is obtained.

$$Pi_D + [(r_DL_{FF}+r_FL_{DD})/(L_{DD}L_{FF}-L_{FD}^2)] \; i_D = -v_FL_{FD}/(L_{DD}L_{FF}-L_{FD}^2) \tag{5.33}$$

and from equation (5.33),

$$T''_{do} = \frac{L_{DD}L_{FF} - L_{FD}^2}{r_DL_{FF} + r_FL_{DD}} \tag{5.34}$$

Direct Axis short circuit subtransient time constant T''_d :

From equations (5.9) and (5.11) after substituting the values of Ψ_F and Ψ_D from equations (5.3) and (5.5), the field and the damper d-axis winding voltages are

$$\sqrt{3}\ L_{mF}Pi_d + L_{FF}Pi_F + L_{FD}Pi_D + r_F i_F = - v_F \tag{5.35}$$

$$\sqrt{3}\ L_{mD}Pi_d + L_{FD}Pi_F + L_{DD}Pi_D + r_D i_D = 0 \tag{5.36}$$

At time $t = 0^+$, $\Psi_F = 0$ as the voltage is suddenly applied.

Neglecting the stator winding resistance from the equation (5.8), and assuming $\Psi_d = 0$ since stator windings are short circuited, we get from equations (5.1) and (5.3),

$$L_d i_d + \sqrt{3}\ L_{mF} i_F + \sqrt{3}\ L_{mD} i_D = 0 \tag{5.37}$$

$$\sqrt{3}\ L_{mF} i_d + L_{FF} i_F + L_{FD} i_D = 0 \tag{5.38}$$

From equations (5.37) and (5.38),

$$\begin{bmatrix} i_d \\ i_F \end{bmatrix} = \frac{1}{(L_{FF}L_d - \sqrt{3}L_{mF}\sqrt{3}L_{mF})} \begin{bmatrix} L_{FF}\sqrt{3}L_{mD} - \sqrt{3}L_{mF}L_{FD} \\ -\sqrt{3}L_{mF}\sqrt{3}L_{mD} + L_{FD}L_d \end{bmatrix} \tag{5.39}$$

From equation (5.36)

$$Pi_F = -\frac{1}{L_{FD}}\left[\sqrt{3}\ L_{mD}Pi_d + L_{DD}Pi_D + r_D i_D\right] \tag{5.40}$$

Substituting the equation (5.40) in the equation (5.35) and the value of i_F from (5.39), the following differential equation is obtained.

$$[(L_{FF}L_{DD}-L_{FD}^2)(L_{FF}L_d-(\sqrt{3}\,L_{mF})^2)+(L_{FF}\sqrt{3}L_{mD}-\sqrt{3}L_{mF}L_{FD})(\sqrt{3}L_{mF}L_{FD}$$
$$-L_{FF}\sqrt{3}\,L_{mD})]\,Pi_D\;+[\,r_DL_{FF}(L_{FF}L_d-(\sqrt{3}L_{mF})^2)-r_FL_{FD}(\sqrt{3}L_{mF}\sqrt{3}L_{mD}$$
$$-L_{FF}L_d)]\;i_D \quad = v_FL_{FD}(L_{FF}L_d-(\sqrt{3}\,L_{mF})^2) \tag{5.41}$$

From equation (5.41),

$$T_d'' = \frac{L_{FF}[\,L_d(L_{FF}L_{DD}-L_{FD}^2)-L_{FF}(\sqrt{3}L_{mD})^2+2\sqrt{3}L_{mD}\sqrt{3}L_{mF}L_{FD}-(\sqrt{3}L_{mF})^2L_{DD}\,]}{r_DL_{FF}(L_{FF}L_d-(\sqrt{3}\,L_{mF})^2)-r_FL_{FD}\,(\sqrt{3}\,L_{mF}\sqrt{3}L_{mD}-L_{FF}L_d)} \tag{5.42}$$

Assuming r_F to be negligible in equation (5.34),

$$T_d'' = \frac{L_d''}{L_d'}\,T_{do}'' \tag{5.43}$$

Similarly, the quadrature-axis open circuit and short-circuit subtransient time constants can be obtained which are as follows

$$T_{qo}'' = \frac{L_{QQ}L_{GG}-L_{GQ}^2}{r_QL_{GG}+r_GL_{QQ}} \tag{5.44}$$

and,

$$T_q'' = \frac{L_q''}{L_q'}\,T_{qo}'' \tag{5.45}$$

Further, the voltage behind the sbutransient reactances and their rate of change have been obtained as follows.

From equation (5.15),

E_q'' = The voltage behind q-axis subtransient reactance

$$= \frac{\omega\Psi_F}{\Delta 1}(\sqrt{3}\,L_{mF}L_{DD}-\sqrt{3}\,L_{mD}L_{FD}) + \frac{\omega\Psi_D}{\Delta 1}(\sqrt{3}L_{mD}L_{FF}-\sqrt{3}L_{mF}L_{FD}) \tag{5.46}$$

Differentiating equation (5.46) w.r.t. time, and substituting $P\Psi_F$ and $P\Psi_D$ from equations (5.9) and (5.11), we get,

$$PE_q'' = -\frac{\sqrt{3}\omega L_{mF}(1-L_{mD}L_{FD}/L_{mF}L_{DD})\left(\frac{V_F}{r_F}+i_F\right)}{\frac{L_{FF}L_{DD}-L_{FD}^2}{r_F L_{DD}}} + \frac{\sqrt{3}\omega L_{mD}i_D(1-L_{mF}L_{FD}/L_{mD}L_{FF})}{\frac{L_{FF}L_{DD}-L_{FD}^2}{r_D L_{FF}}} \tag{5.47}$$

Substituting for $L_{FF}L_{DD} - L_{FD}^2$ from equation (5.34) in (5.47), we get

$$PE_q'' = -\frac{(E_{FD}+E_q')K1}{T_{do}''\left(1+\frac{T_{do}'}{T_D}\right)} + \frac{K2\quad E_{qD}''}{T_{do}''\left(1+\frac{T_D}{T_{do}'}\right)} \tag{5.48}$$

where, $K1 = 1-L_{mD}L_{FD}/L_{mF}L_{DD}$

$K2 = 1-L_{mF}L_{FD}/L_{mD}L_{FF}$

$T_D = L_{DD}/r_D$

Similarly, from equation (5.18),

$$E_d'' = \frac{\sqrt{3}\omega\Psi_G}{L_{GG}L_{QQ}-L_{GQ}^2}(L_{mG}L_{QQ}+L_{mQ}L_{GQ}) + \frac{\sqrt{3}\omega\Psi_Q}{L_{GG}L_{QQ}-L_{GQ}^2}(L_{mG}L_{GQ}+L_{mQ}L_{GG}) \tag{5.49}$$

Differentiating the above equation w.r.t time and substituting the values of $P\Psi_G$ and $P\Psi_Q$ from (5.10) and (5.12), we have,

$$PE_d'' = -\frac{\sqrt{3}\omega L_{mG}(1+L_{mQ}L_{QG}/L_{mG}L_{QQ})I_G}{(L_{GG}L_{QQ}-L_{GQ}^2)/L_{QQ}r_G} - \frac{\sqrt{3}\omega L_{mQ}(1+L_{mG}L_{GQ}/L_{mQ}L_{GG})I_G}{(L_{QQ}L_{GG}-L_{GQ}^2)/r_Q L_{GG}} \tag{5.50}$$

Introducing the equation (5.44) in the equation (5.50), we have

$$PE_d'' = -\left[\frac{K3\ E_d}{T_{qo}''\left(1+\frac{T_{qo}'}{T_Q}\right)} + \frac{K4\ E_{dQ}}{T_{qo}''\left(1+\frac{T_Q}{T_{qo}'}\right)}\right] \tag{5.51}$$

where,

$$K3 = 1 + L_{mQ}L_{GQ}/L_{mG}L_{QQ}$$

$$K4 = 1 + L_{mG}L_{GQ}/L_{mQ}L_{GG}$$

$$T_Q = L_{QQ}/r_Q$$

Equations (5.48) and (5.51) give the rate of change of voltage behind q and d-axis subtransient reactances.

when the d- and q-axis damper windings are not considered, the transient state is approached, and the equations (5.47) and (5.50) reduce to

$$PE_q' = -\frac{1}{T_{do}'}(E_{FD} + E_q) \tag{5.52}$$

and

$$PE_d' = -\frac{1}{T_{qo}'} E_d \tag{5.53}$$

5.3 A CASE STUDY

The transient stability investigation has been carried out on a sample system as shown in Fig. 5.1 for the different system configurations in order to compare the performance of a mixed three-phase and six- phase system with that of a completely three-phase system. Also, a completely six-phase system has been compared with the conventional three-phase system.

5.3.1 Sample Network

A nine bus three-machine sample network [55] has been taken for transient stability investigation. The line and the transformer data are provided in Appendix B; the synchronous machine data in Table 5.1. Prefault conditions for the different configurations with the same loading conditions are given in chapter 4. The load flow data for

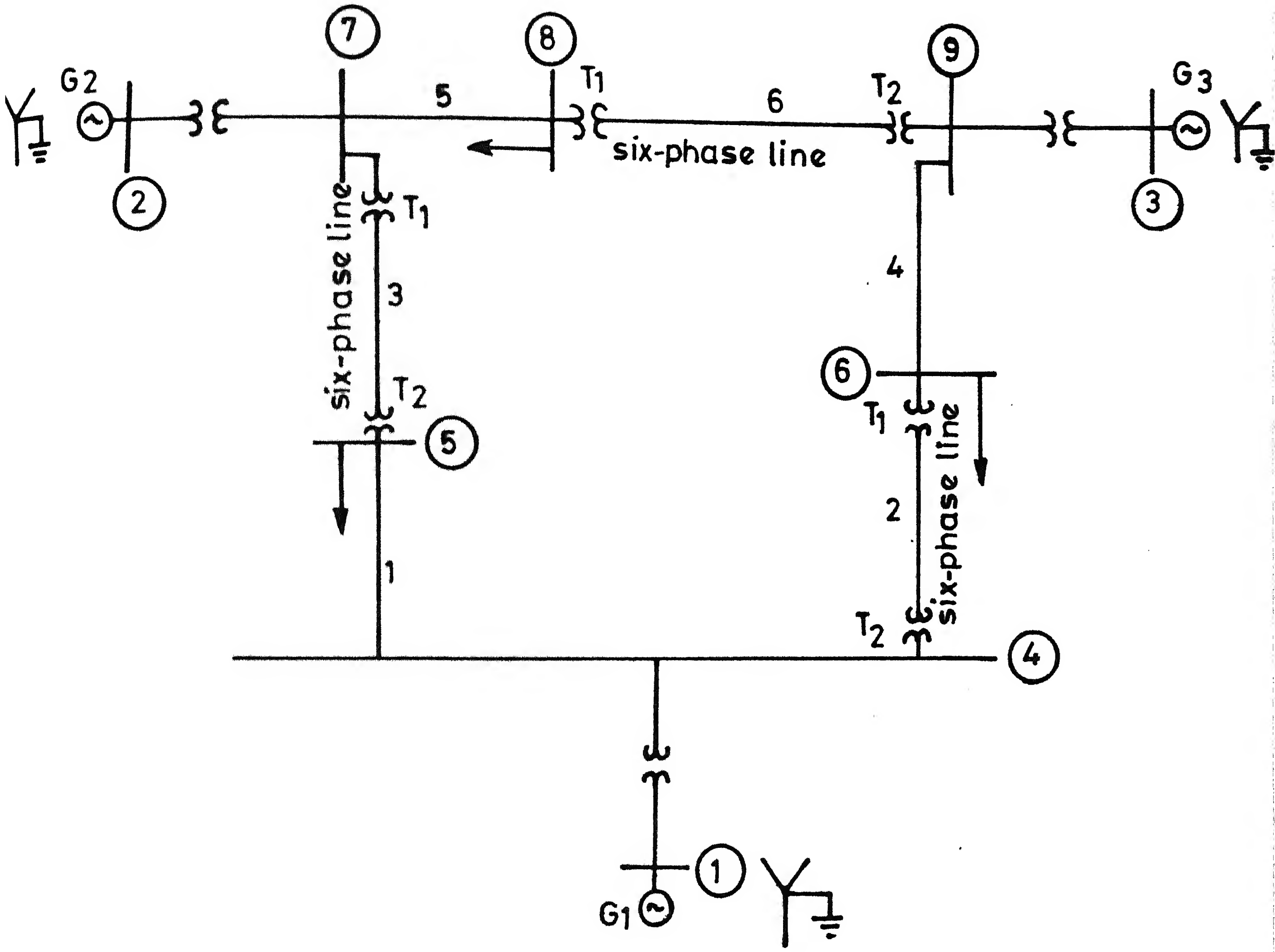


FIG.5.1 Nine-bus three machine sample system for the transient stability Investigation; Line nos. 2, 3 and 6 are six phase lines.

the increased loading has been shown in Table 5.2. The pre-fault voltage for the increased loading for a mixed system, and for a completely six-phase system with the same loading and with increased loading have been provided in Table 5.3.

5.3.2 System Representation

i) Machine Representation :

Synchronous machines have been represented by the classical model where the changes in flux linkages and the saliency are neglected. Thus, the synchronous machine is represented by a constant voltage source behind the direct-axis transient reactance for the purpose of this study.

ii) Network Representation :

The network performance equations employing single-phase equivalent representation of various elements are described as

$$YV = I \tag{5.54}$$

The solution of equation (5.54) is obtained by Gauss-Siedel iterative technique.

iii) Load Representation:

The load is modelled as a constant impedances from bus to ground.

iv) Swing equation :

The equations for electro-mechanical motion of the machine employing the simplified model are

$$\frac{d\delta_i}{dt} = \omega_i(t) - 2\pi f \tag{5.55}$$

$$\frac{d\omega_i}{dt} = \frac{\pi f}{H_i}(P_{mi} - P_{ei}(t)) \tag{5.56}$$

where, $i=1,\ldots,m$; $m = 3$, the number of machines in operation the effect of damping and governor action has been neglected in this model.

5.3.3 Sample Network with Different Configurations :

The different configurations under transient stability investigations are : 1) a completely three- phase system, Line No. 2,3 and 6 have been assumed as double-circuit three-phase lines; 2) a mixed three- phase and six-phase systems where the six-phase lines are obtained by converting double-circuit three-phase lines; three cases have been considered, for example, a) when only line no. 2 is converted, b) when two lines (nos. 2 and 3) are converted and c) when all the double-circuit three-phase lines (nos. 2,3, and 6) are converted ; 3) a completely six-phase system where the single-circuit as well as the double circuit three-phase lines are converted to single circuit six-phase lines.

When a three-phase line is converted to a six-phase line, two voltages have been considered for the study of mixed three-phase and six-phase systems with the same loading as that of the completely three-phase systems; in one case the line to ground voltage of the six-phase line is $\sqrt{3}$ times that of the double circuit three-phase line, and in the second case, the line to ground voltage of the two lines (i.e., three-phase and the six-phase) is assumed to be equal. If the power system is assumed to meet the increased demand, say by $\sqrt{3}$, then with this increased system loading, the line to line voltages of the two systems have been assumed to be equal. For a completely six-phase system, the transient stability has been investigated with the same loading as the original three-phase system, and also with the increased loading by assuming the same phase to ground voltage of the two systems. The system has been assumed to be balanced and therefore, the study has been conducted based on one line diagram.

As the conversion requires one three-phase/six-phase transformer at each end of the six-phase line, the single-phase equivalent of a six-phase line integrated with two transformers has been calculated with 8% leakage impedance of the transformer from equation (4.12) and (4.13).

When the system becomes completely six-phase, it requires six-phase trasformers in place of three-phase transformers; in this case, the leakage impedances of the six-phase transformers have been assumed to be the same as those of three-phase transformers. The six-phase synchronous machines have been assumed to have the same inductances (i.e. inductive coefficients) as those of three-phase machines.

TABLE 5.1

Machine Data on 100 MVA Base of the Sample System Shown in Fig. 5.1

Machine No.	connecting Bus	Inertia constant (Sec.)	Direct-Axis Trans. reactance
1	1	23.64	(1) .0608
			(3) .1216
2	2	6.4	(1) .1198
			(3) .2396
3	3	3.01	(1) .1813
			(3) .3626

In the last column (1) and (3) refer to the direct-axis transient reactance for three-phase and 6-phase machines respectively.

TABLE 5.2

Assumed Bus Voltages for the Load Flow Analysis of the Sample System Shown in Fig. 5.1 for increased Loading

Bus No.	Assumed bus voltage	Generation		Load	
		MW	MVAR	MW	MVAR
1	1.04+j0.0	0.0	0.0	0.0	0.0
2	1.00+j0.0	163.0	6.7	0.0	0.0
3	1.00+j0.0	85.0	-10.9	0.0	0.0
4	1.00+j0.0	0.0	0.0	0.0	0.0
5	1.00+j0.0	0.0	0.0	216.5	86.6
6	1.00+j0.0	0.0	0.0	156.0	52.0
7	1.00+j0.0	0.0	0.0	0.0	0.0
8	1.00+j0.0	0.0	0.0	173.0	60.6
9	1.00+j0.0	0.0	0.0	0.0	0.0

TABLE 5.3

Bus Voltages of the Sample System Shown in Fig. 5.1

Bus No.	Bus Voltage (1) when line nos 2,3, & 6 are converted to six-phase lines		(2) for a completely six-phase with same loading		with increased loading	
1	1.04	0.0	1.04	0.0	1.04	0.0
2	1.078	-8.174	1.1563	4.15	1.0107	-4.223
3	1.0746	-11.605	1.1582	1.6943	1.0198	-6.6475
4	1.0464	-9.212	1.0698	-1.0496	1.0237	-4.7036
5	1.0369	-15.863	1.0768	-2.3654	.9905	-9.192
6	1.0434	-13.867	1.1183	-2.1237	1.0136	-10.225
7	1.0783	-13.2016	1.1553	1.9645	1.0099	-7.0835
8	1.0687	-16.471	1.1536	.4756	.9975	-9.87
9	1.0815	-14.0615	1.1612	.6331	1.0232	-8.0152

Slack bus generation (I) = (302.47, 12.74)

Slack bus generation (II)= (70.76, -107.0)

with same loading

Slack bus generation with increased loading = (303.14,71.16)

5.3.4 Result and Discussions

A symmetrical (n-phase to ground) fault is assumed to occur at the buses 4, 7 and 9, one at a time for the different power system configurations under various post-fault conditions. Graphs (Machine angle difference vs. time) have been plotted as shown in Figs. 5.2 - 5.7 for 3 cases : I) a completely three-phase system II) a mixed three-phase and six-phase system under voltage conditions A; III) a completely six-phase system under voltage condition B; the same system loading has been considered. The machine angle difference vs. time for the fault at the buses 4,7 and 9 are observed to compare the performance affected by the conversion of double-circuit three-phase lines to six-phase lines, and also to study the impact of converting a three-phase system to a completely six-phase system. It is observed from the graphs that there is imporovement in the stability with the six-phase system.

The critical clearing times for the faults at the buses have been shown in Tables 5.4.-5.11. It is observed from the tables that when a double circuit three-phase line is converted to a single circuit six-phase line with the same line to line (adjacent phase) voltage, the critical clearing time increases as the number of conversions increases

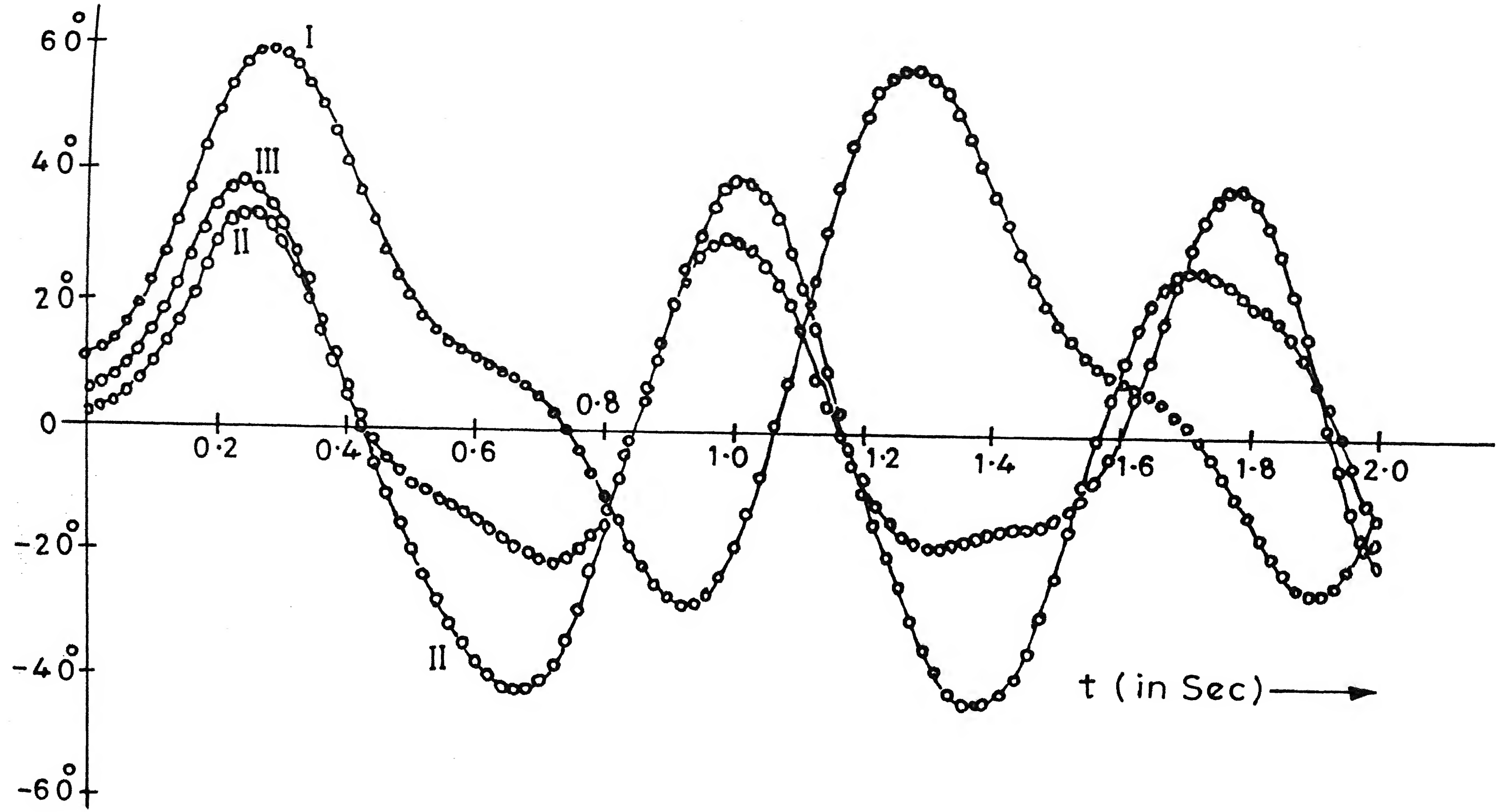


FIG. 5.3 Plot of $(\delta_3 - \delta_1)$ Vs time for the fault near bus 4.

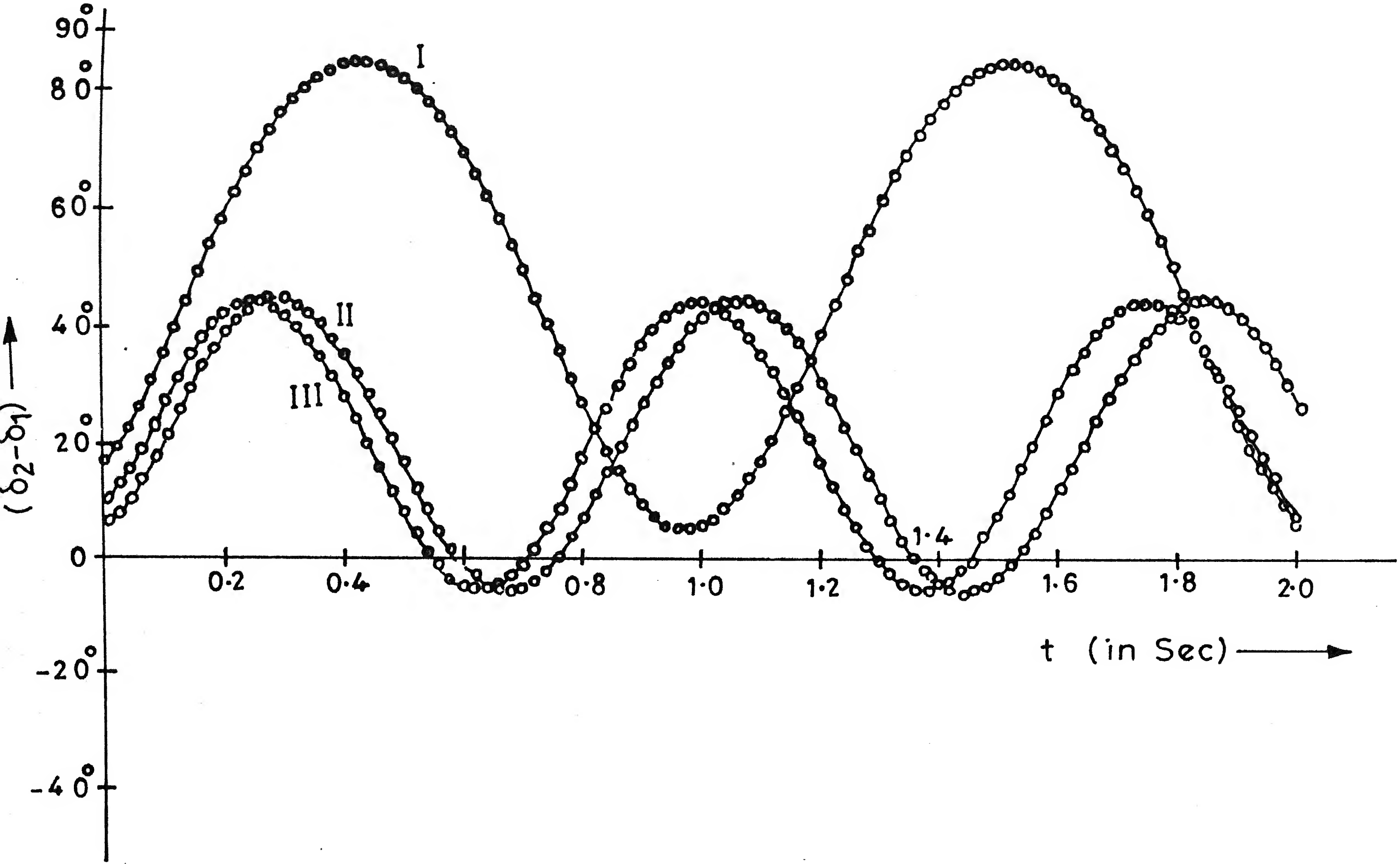


FIG. 5.4 Plot of $(\delta_2-\delta_1)$ Vs time for the fault near bus 7. The fault is cleared in 0.8 Sec. by opening line 3

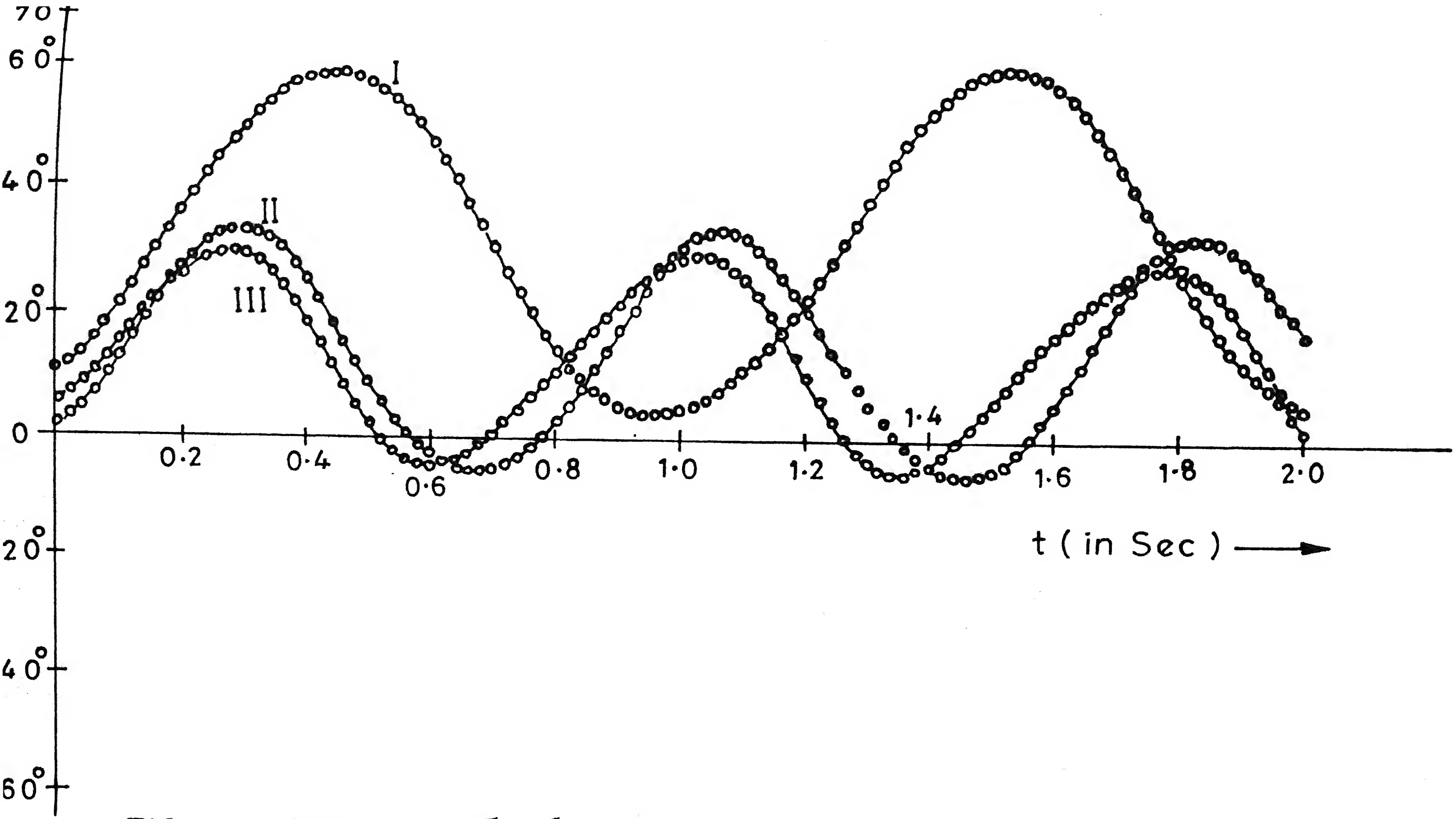


FIG. 5.5 Plot of $(\delta_3 - \delta_1)$ Vs time for the fault near bus 7 .

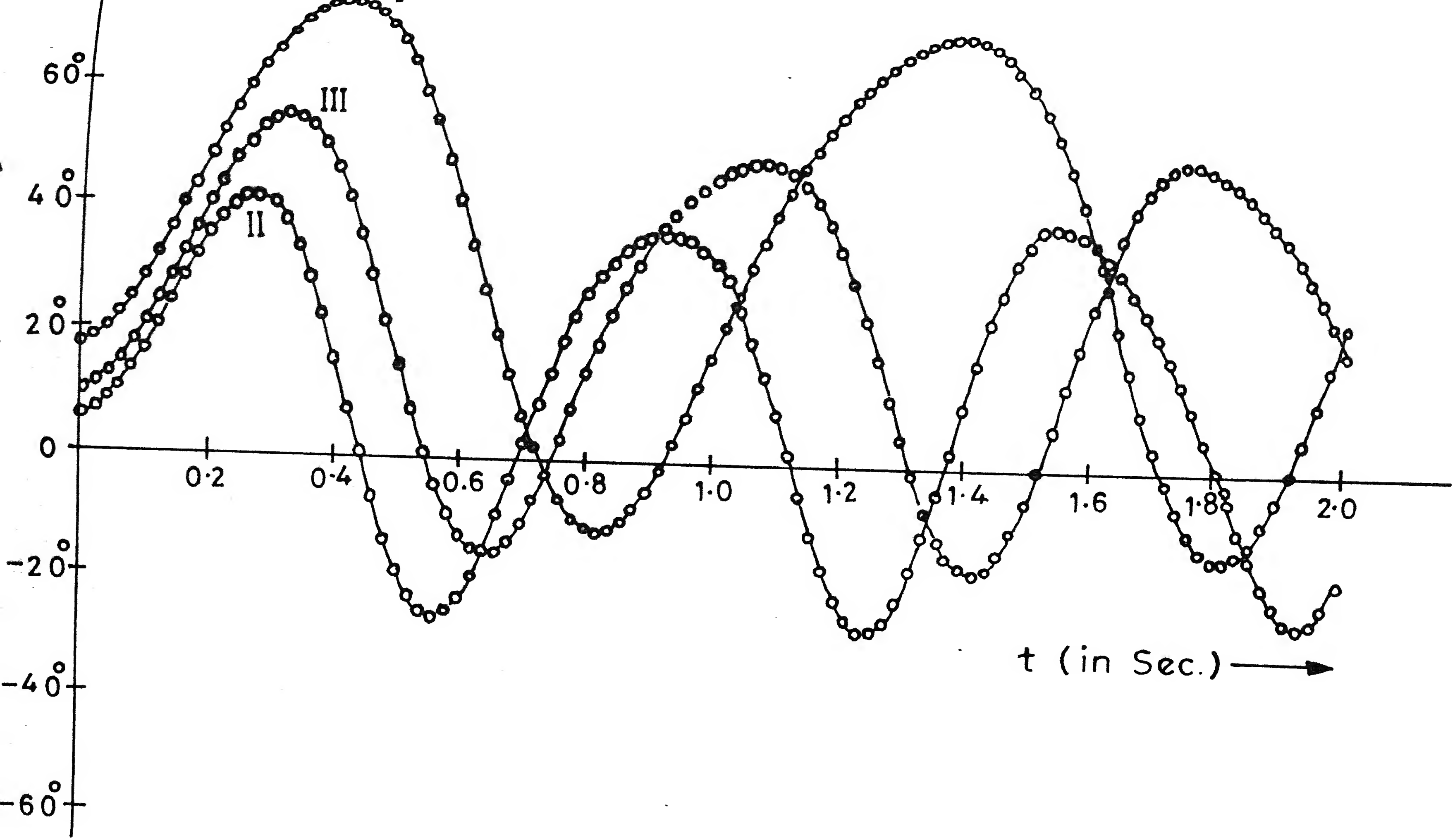


FIG.5.6 Plot of $(\delta_2-\delta_1)$ Vs time for the fault near bus 9. The fault is cleared in 0.14 Sec. by opening line 4.

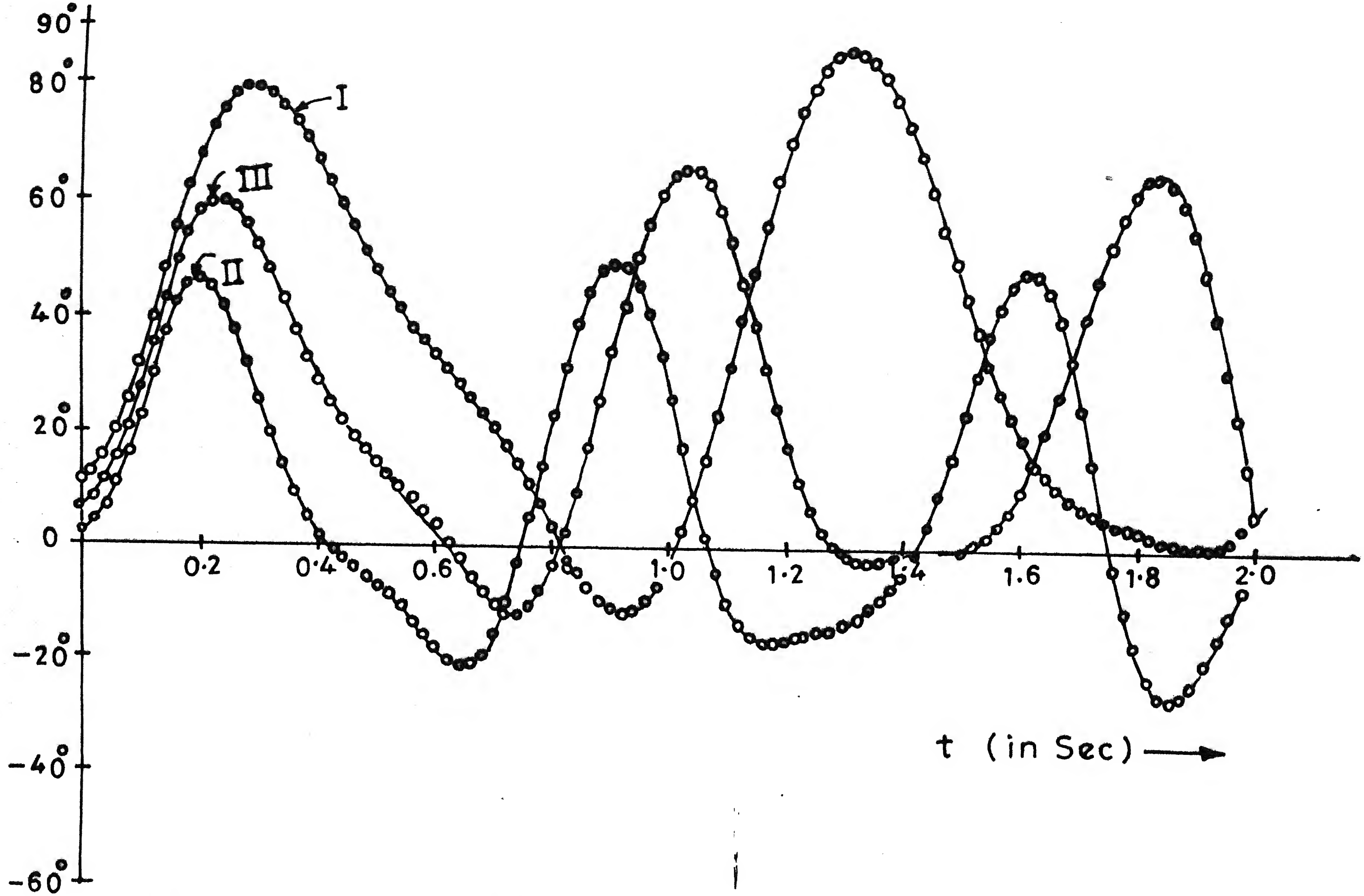


FIG.5.7 Plot of $(\delta_3 - \delta_1)$ Vs time for the fault near bus 9.

irrespective of the fault locations and the post-fault configurations. But when the power is transmitted at the same phase to ground voltage as that of the three-phase line, it is observed from Table 5.5 that, when line no. 2 is converted to six-phase line, the critical clearing time for different post-fault conditions are the same. For example, when the fault near the bus 4 is cleared by tripping line 2, which is a six-phase line, the critical clearing times for the two configurations are .24 sec. - .25 sec.

When line nos. 3 and 6 are converted to six-phase lines, it is observed that the critical clearing time is less when the line no.1 is tripped, and the critical clearing time under configurations II and III are the same, i.e., .21 - .22 Sec. It means that the conversion of line no.3 reduces the stability and line no. 6 does not affect it when the fault is at bus no.4. Therefore, it can be concluded that, whether the conversion to six-phase system reduces the stability or not depends upon the location of that line. From Tables 5.7 and 5.9, which provide the clearing times for the fault at buses 7 and 9 respectively, the critical clearing time increases and decreases as the power is transmitted at the same adjacent phase voltages or at the same phase to ground voltage for the same loading. Further, the critical

TABLE 5.4

Critical Clearing Time for the Three-Phase to Ground Fault near the different Buses of a Completely Three-Phase Sample Network

Faulted bus	Post fault condition	cct (in Sec.)
4	a) Fault removed	.25 - .26
	b) Line 1 is tripped	.22 - .23
	c) Line 2 is tripped	.24 - .25
7	a) Fault removed	.16 - .17
	b) Line No. 3 is tripped	.09 - .10
	c) Line No. 5 is tripped	.14 - .15
9	a) Fault removed	.19 - .20
	b) Line No. 4 is tripped	.16 - .17
	c) Line No. 6 is tripped	.17 - .18

TABLE 5.5

Critical Clearing Time for the n-phase to Ground fault at the bus No.4 of the Sample System for Case a

System config.	Post fault conditions	cct (in Sec.) Voltage condition A	B
I	a) fault removed	.26 - .27	.25 - .26
	b) line 1 is tripped	.23 - .24	.22 - .23
	c) line 2 is tripped	.24 - .25	.24 - .25
II	a) fault removed	.29 - .30	.23 - .24
	b) line 1 is tripped	.27 - .28	.20 - .21
	c) line 2 is tripped	.29 - .30	.21 - .22
III	a) fault removed	.34 - .35	.22 - .23
	b) line 1 is tripped	.32 - .33	.19 - .20
	c) line 2 is tripped	.33 - .34	.21 - .22

Voltage configuration :

A refers to the mixed system when line to line (i.e adjacent line) voltage of six-phase line and the double circuit three-phase lines are the same.

B refers to the mixed system when the six-phase line and the double circuit three-phase lines have the same phase to ground voltage.

TABLE 5.6

Critical Clearing Time for the n-Phase to Ground Fault at Bus No.4 of the Sample System for Case b

System config.	Post fault condition	cct(in Sec.) Voltage condition A	B
I	a) fault removed	.21 - .22	.25 - .26
	b) line 1 is tripped	.19 - .20	.22 - .23
	c) line 2 is tripped	.20 - .21	.24 - .25
II	a) fault removed	.18 - .19	.24 - .25
	b) line 1 is tripped	.14 - .15	.21 - .22
	c) line 2 is tripped	.17 - .18	.23 - .24
III	a) fault removed	.16 - .17	.24 - .25
	b) line 1 is tripped	.12 - .13	.21 - .22
	c) line 2 is tripped	.15 - .16	.23 - .24

TABLE 5.7

Critical Clearing Time for the n-Phase to Ground Fault at Bus No.7 of the Sample System for Case a

System config.	Post fault condition	cct (in Sec.) Voltage condition A	B
I	a) fault removed	.17 - .18	.16 - .17
	b) line 3 is tripped	.12 - .13	.07 - .08
	c) line 5 is tripped	.15 - .16	.14 - .15
II	a) fault removed	.20 - .21	.15 - .16
	b) line 3 is tripped	.16 - .17	.05 - .06
	c) line 5 is tripped	.19 - .20	.09 - .10
III	a) fault removed	.22 - .23	.15 - .16
	b) line 3 is tripped	.19 - .20	0.0 - 0.0
	c) line 5 is tripped	.21 - .22	.08 - .09

TABLE 5.8

Critical Clearing Time for the n-Phase to Ground Fault at Bus No.7 of the Sample System for Case b

System config.	Post fault condition	cct (in Sec.) Voltage condition A	B
I	a) fault removed	.15 - .16	.17 - .18
	b) line 3 is tripped	.05 - .06	.07 - .08
	c) line No. 5 is tripped	.11 - .12	.14 - .15
II	a) fault removed	.14 - .15	.16 - .17
	b) line no. 3 is tripped	0.0 -	.08 - .09
	c) line no.5 is tripped	.12 - .13	.11 - .12
III	a) fault removed	.12 - .13	.16 - .17
	b) line no.3 is tripped	0.0	.06 - .07
	c) line no. 5 is tripped	.09 - .10	.11 - .12

TABLE 5.9

Critical Clearing Time for the n-Phase to Ground Fault at Bus No. 9 of the Saple System for Case a.

System config.	Post fault condition	cct (in Sec.) Voltage condition A	B
I	a) fault rem oved	.19 - .20	.19 - .20
	b) line 4 is tripped	.17 - .18	.16 - .17
	c) line 6 is tripped	.18 - .19	.17 - .18
II	a) fault removed	.21 - .22	.18 - .19
	b) line 4 is tripped	.20 - .21	.13 - .14
	c) line 6 is tripped	.20 - .21	.17 - .18
III	a) fault removed	.22 - .23	.18 - .19
	b) line 4 is tripped	.22 - .23	.11 - .12
	c) line 6 is tripped	.21 - .22	.17 - .18

TABLE 5.10

Critical Clearing Time for the n-Phase to Ground Fault at Bus No. 9 of the Sample System for Case b

System config.	Post fault condition	cct (in Sec.) for different voltage condi. A	B
I	a) fault removed	.18 - .19	.19 - .20
	b) line no.4 is tripped	.13 - .14	.16 - .17
	c) line 6 is tripped	.16 - .17	.17 - .18
II	a) fault removed	.16 - .17	.19 - .20
	b) line no. 4 is tripped	.11 - .12	.15 - .16
	c) line no. 6 is tripped	.14 - .15	.17 - .18
III	a) fault removed	.15 - .16	.18 - .19
	b) line no. 4 is tripped	.10 - .11	.13 - .14
	c) line no. 6 is tripped	.13 - .14	.17 - .18

TABLE 5.11

Critical Clearing Time for n-Phase to Ground Fault with Increased System Loading for Case a

Faulted Bus	Post fault condition	cct(in Sec.) System configuration: when line nos.2,3&6 are 6-phase lines (Volt. config.) A	For a completely 6-phase system (volt. config. B) same loading	increased loading
4	1) fault removed	.77 - .78	.33 - .34	1.54-1.55
	2) line no.1 is tripped	.47 - .48	.32 - .33	1.19-1.2
	3) line no.2 is tripped	.60 - .61	.32 - .33	1.24-1.25
7	1) fault removed	.27 - .28	.20 - .21	.26-.27
	2) line no.3 is tripped	.26 - .27	.18 - .19	.24-.25
	3) line no.5 is tripped	.28 - .29	.18 - .19	.24-.25
9	1) fault removed	.28 - .29	.21 - .22	.26-.27
	2) line no.4 is tripped	.27 - .28	.19 - .20	.25-.26
	3) line no.6 is tripped	.27 - .28	.20 - .21	.25-.26

clearing times for the fault at the different buses with the help of prefault conditions obtained for case b are provided in Tables 5.6, 5.8 and 5.10. It is observed from the Tables that the critical clearing times are less under both transmission voltages; however, the critical clearing times are more in case when the six-phase lines have the same phase to ground voltage as the three-phase line.

In Table 5.11, the critical clearing times for the increased loading i.e., $\sqrt{3}$ times that of the completely three-phase system, have been tabulated for the different cases : 1) when lines 2, 3 and 6 are converted to six-phase lines and the power is transmitted at the same line to line (adjacent phase) voltage of the two systems and (2) a completely six-phase system; case 2 has been considered for two loading conditions with phase to ground voltage equal to that of three-phase system.

It is observed from the comparision of Tables 5.5, 5.7, 5.9 and 5.11 that the critical clearing times for the mixed system with increased loading are higher with those having the same loading. For a completely six-phase system with the same loading as well as with the increased loading, the critical clearing time for the two loadings are more than that for the completely three-phase system. It is further

observed that the critical clearing times for a completely six-phase system are higher with increased loading than with the same loading as the completely three-phase system.

5.4 CONCLUSION

The direct- and the quadrature- **axis** transient and subtransient inductances as well as the different time constants of a six-phase synchronous machine have been calculated which will be useful in analyzing the six-phase machine as well as the power system network wher ever it is incorporated. The transient stability has been investigated for the different power system configurations especially with a view to study the impact of converting double circuit three-phase lines to single circuit six-phase lines. The results reveal the following observations.

1. If the phase voltage i.e. line to neutral voltage of the six-phase line is equal to $\sqrt{3}$ times that of the double circuit three-phase line, the stability increases with the same loading as well as with increased loading.
2. If the uprating of the double-circuit three-phase lines are carried out on the same loading with equal phase (i.e. line to neutral) voltage, the stability reduces.

3. If a three-phase system is converted to a completely six-phase system , the stability of the six-phase system increases with the same loading as well as with increased loading, when the phase voltages of the two systems are equal.

4. The impact of converting a double-circuit three-phase line to a single-circuit six-phase line on the stability depends on the location of the converted line from the fault.

5. If the power is transmitted through six-phase line having the adjacent voltages equal to those of double-circuit three-phase lines, the stability of the mixed system is more with increased loading than with the same loading as that of the completely three-phase system, and also, a completely six-phase system is more stable with increased loading than with the same loading when both the systems have the same phase to ground voltage.

CHAPTER 6

DYNAMIC STABILITY ANALYSIS OF SIX-PHASE SYSTEMS

6.1 INTRODUCTION

In the previous chapter the transient stability investigation has been carried out on a sample network for its different configurations in order to study the impact of converting double circuit three-phase lines to single-circuit six-phase lines on the transient stability. This chapter presents an eleborate study of dynamic stability of the multi-phase (six-phase) system. For the dynamic stability study, various mathematical models of three-phase synchronous machines are available [55,56,69]. A mathematical model of the multi-machine power network has been provided in section 6.2 for the present study. A linearized model of the m-port reduced network has been given in section 6.2.1, and in section 6.2.2 a linearized model of a power system network has been provided incorporating the machine details. A case study for dynamic stability investigation is conducted in section 6.3. A sample system for the purpose of study has been shown in section 6.3.1. The various system configurations of the sample network have been discussed in section 6.3.2. Further, the results of the case study have been

presented and discussed in section 6.3.3. Finally, this chapter concludes in section 6.4 based on the discussions of the results of investigations.

6.2 DYNAMIC MODEL OF THE MULTIMACHINE POWER SYSTEM

Various models, such as two-axis model, one-axis model, classical model etc. for dynamic stability study of multi-machine system are available in the literature[55,70,71]. The consideration of a particular model for the study depends upon the availability of data and also to what extent the accuracy is desired in the analysis. Here, for the present study, the two-axis model and the classical model for a synchronous machine has been considered. The voltage and the current relations of machines are presented. The linearized model for the power system network has been given.

6.2.1 Linearized Model of Power System Network

The linearized model of a reduced power system network (Fig. 6.1) can be expressed by a set of equations as given below (Appendix C).

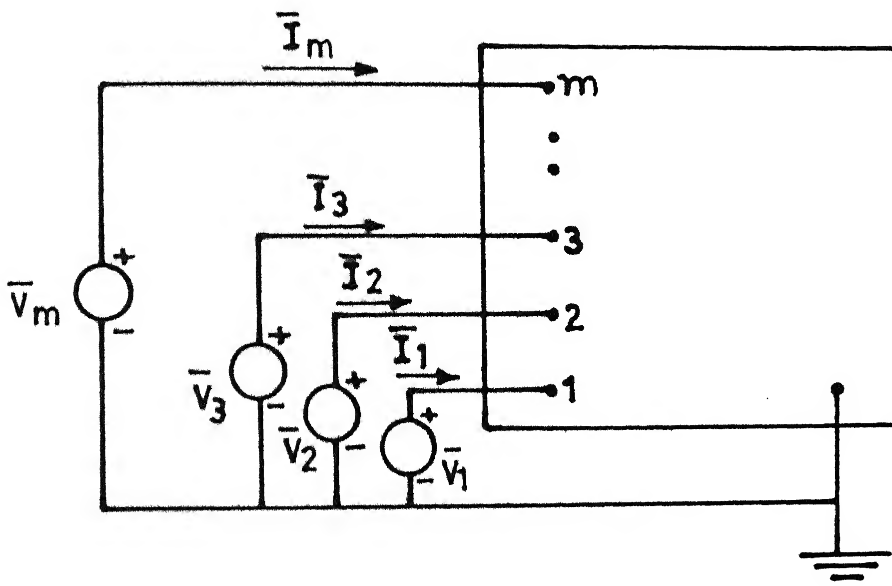


FIG. 6.1 A Reduced m-port Network .

$$\begin{bmatrix} \Delta \bar{I}_1 \\ \Delta \bar{I}_2 \\ \cdots \\ \Delta \bar{I}_m \end{bmatrix} = \begin{bmatrix} Y_{11} e^{j\theta_{11}} & \cdots & Y_{1m} e^{j(\theta_{1m}-\delta_{1mo})} \\ Y_{21} e^{j(\theta_{21}-\delta_{210})} & \cdots & Y_{2m} e^{j(\theta_{2m}-\delta_{2mo})} \\ \cdots & \cdots & \cdots \\ Y_{m1} e^{j(\theta_{m}-\delta_{m10})} & \cdots & Y_{mm} e^{j\theta_{mm}} \end{bmatrix} \begin{bmatrix} \Delta \bar{V}_1 \\ \Delta \bar{V}_2 \\ \cdots \\ \Delta \bar{V}_m \end{bmatrix}$$

$$-j \begin{bmatrix} \sum_{K=1}^{m} \bar{V}_{KO} \, Y_{1K} e^{j(\theta_{1K}-\delta_{1KO})} \, \Delta \delta_{1K} \\ \sum_{K=1}^{m} \bar{V}_{KO} \, Y_{2K} e^{j(\theta_{2K}-\delta_{2KO})} \, \Delta \delta_{2K} \\ \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ \sum_{K=1}^{m} \bar{V}_{KO} \, Y_{mK} e^{j(\theta_{mK}-\delta_{mKO})} \, \Delta \delta_{mK} \end{bmatrix} \qquad (6.1)$$

6.2.2 Linearized Models of Synchronous Machines

The two models for synchronous machines are presented.

1. <u>Two-axis Model of a Synchronous Machine</u>

In the two axis model, the transients effects are taken into consideration while the subtransient effects are

neglected [55] . The transient effects are dominated by the rotor circuit. The rotor has two circuits; one main field winding on its d-axis and the other equivalent q-axis formed by the solid rotor.

The two-axis model of each machine is given by

$$T'_{qOi}\,\dot{E}'_{di} = -E'_{di} - (x_{qi} - x'_{q})\, I_{qi} \tag{6.2}$$

$$T'_{doi}\,\dot{E}'_{qi} = E_{FDi} - E_{qi} + (x_{di} - x'_{di})\, I_{di} \tag{6.3}$$

$$M_{i}\,\dot{\omega}_{i} = T_{mi} - (\,I_{di}\,E'_{di} + I_{qi}\,E'_{qi}\,) - D_{i}\,\omega_{i} \tag{6.4}$$

$$\dot{\delta}_{i} = \omega_{i} - \omega_{o} \tag{6.5}$$

where $i = 1,2,\ldots,m$; m is the number of machines.

A linear model for synchronous machine has been obtained by linearizing equations (6.2)-(6.5), which are as follows.

$$T'_{qoi}\,\Delta\dot{E}'_{di} = -(x_{qi} - x'_{q})\,\Delta I_{qi} - \Delta E'_{di} \tag{6.6}$$

$$T'_{doi}\,\Delta\dot{E}'_{qi} = \Delta E_{FDi} - \Delta E_{qi} + (x_{di} - x'_{di})\,\Delta I_{di} \tag{6.7}$$

$$M_{i}\,\Delta\dot{\omega}_{i} = \Delta T_{mi} - D_{i}\Delta\omega_{i} - (I_{dio}\,\Delta E'_{di} + E'_{dio}\,\Delta I_{di} + E'_{qio}\,\Delta I_{qi} + I_{qio}\,\Delta E'_{qi}) \tag{6.8}$$

$$\Delta \dot{\delta}_i = \Delta\omega_i \tag{6.9}$$

It is observed from equations (6.6)-(6.9) that each machine represents a fourth order system with state variables $\Delta E'_{qi}$, $\Delta E'_{di}$, $\Delta\omega_i$ and $\Delta\delta_i$, but the unknowns are six, namely ΔI_{qi} and ΔI_{di} in addition to the state variables. These two unknown parameters of each machine can be found from equation (6.1).

In equation (6.1), $\Delta \bar{V}_i$ is increments in internal generator voltages.

2. A Classical Model of a Synchronous Machine

The classical model of the synchronous machine is given by

$$M_i \dot{\omega}_i = T_{mi} - E_i I_{qi} - D_i \omega_i \tag{6.10}$$

$$\dot{\delta}_i = \omega_i - \omega_o \tag{6.11}$$

Thus, the linearized classical model of the synchronous machine from equations (6.10) and (6.11) are

$$M_i \Delta \dot{\omega}_i = \Delta T_{mi} - E_{io} \Delta I_{qi} - D_i \Delta\omega_i \tag{6.12}$$

$$\Delta \dot{\delta}_i = \Delta\dot{\omega}_i \tag{6.13}$$

where $i = 1,2,\ldots,r$; r is the number of machines represented by classical model.

Thus, the machine represented by the classical model will have only $\Delta\omega_r$ and $\Delta\delta_r$ as state variables. In equation (6.12). E_{io} is known and ΔI_{qi} is unknown variable that is eliminated with the help of equation (6.1).

If the machines in the power system network are represented by both two-axis model and the classical model, then in order to eliminate ΔI_{qr} of the classically represented machine, a relation between ΔI_{qr} and the currents of the machines represented by two-axis model is obtained. Also, the voltages ΔV_{qi} and ΔV_{di} of the machines represented by two-axis model are obtained in terms of ΔI_{qi} and ΔI_{di} of these machines, and voltages E_r of the machines represented classically [55] .

The dynamical modelling for the three-machine power system network has been given below.

A Particular Case :

A linear model of a power system network with three synchronous machines has been developed.

Taking linear equations (6.12) - (6.13) for a classical model of machine 1 and equations (6.6) - (6.9) for

a two-axis model of machines 2 and 3, the resulting equations in the matrix form will be

$$\begin{bmatrix} \Delta\dot{\omega}_1 \\ \Delta\dot{E}'_{q2} \\ \Delta\dot{E}'_{d2} \\ \Delta\dot{\omega}_2 \\ \Delta\dot{E}'_{q3} \\ \Delta\dot{E}'_{d3} \\ \Delta\dot{\omega}_3 \\ \Delta\dot{\delta}_{12} \\ \Delta\dot{\delta}_{13} \end{bmatrix} = \begin{bmatrix} \frac{E_1}{M_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_{d2}-x'_2}{T'_{do2}} & 0 & 0 \\ 0 & 0 & -\frac{x_{q2}-x'_2}{T'_{qo2}} & 0 & 0 & 0 \\ 0 & 0 & -\frac{E'_{q20}}{M_2} & -\frac{E'_{d20}}{M_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{x_{d3}-x'_3}{T'_{do3}} \\ 0 & 0 & 0 & 0 & \frac{x_{q3}-x'_3}{T'_{qo3}} & 0 \\ 0 & 0 & 0 & 0 & -\frac{E'_{q30}}{M_3} & -\frac{E'_{d30}}{M_3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta I_{q1} \\ \Delta I_{d1} \\ \Delta I_{q2} \\ \Delta I_{d2} \\ \Delta I_{q3} \\ \Delta I_{d3} \end{bmatrix}$$

$$
+\begin{bmatrix}
-\frac{D_1}{M_1}\Delta\omega_1 \\
-\frac{1}{T'_{do2}}\Delta E'_{q2} \\
-\frac{1}{T'_{qo2}}\Delta E'_{d2} \\
-\frac{D_2}{M_2}\Delta\omega_2 - \frac{1}{M_2} I_{q20}\Delta E'_{q2} - \frac{1}{M_2} I_{d20}\Delta E'_{d2} \\
-\frac{1}{T'_{do3}}\Delta E'_{q3} \\
-\frac{1}{T'_{qo3}}\Delta E'_{d3} \\
-\frac{D_3}{M_3}\Delta\omega_3 - \frac{1}{M_3} I_{q30}\Delta E'_{q3} - \frac{1}{M_3} I_{d30}\Delta E'_{d3} \\
\Delta\omega_1 - \Delta\omega_2 \\
\Delta\omega_1 - \Delta\omega_3
\end{bmatrix}
+
\begin{bmatrix}
\frac{1}{M_1}\Delta T_{m1} \\
\frac{1}{T'_{do2}}\Delta E_{FD2} \\
0 \\
\frac{1}{M_2}\Delta T_{m2} \\
\frac{1}{T'_{do3}}\Delta E_{FD3} \\
0 \\
\frac{1}{M_3}\Delta T_{m3} \\
0 \\
0
\end{bmatrix}
\tag{6.14}
$$

From equation (6.1) with $\overline{V}$ replaced by $\overline{E}'$ and for a three-machine system (using $\delta_{21} = -\delta_{12}$, $\delta_{31} = -\delta_{13}$), we have

$$\begin{bmatrix} \Delta \bar{I}_1 \\ \Delta \bar{I}_2 \\ \Delta \bar{I}_3 \end{bmatrix} = \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j(\theta_{12}-\delta_{120})} & Y_{13}e^{j(\theta_{13}-\delta_{130})} & -j\bar{E}'_{20}Y_{12}e^{j(\theta_{12}-\delta_{120})} & -j\bar{E}'_{30}Y_{13}e^{j(\theta_{13}-\delta_{130})} & 0 \\ Y_{21}e^{j(\theta_{21}-\delta_{210})} & Y_{22}e^{j\theta_{22}} & Y_{23}e^{j(\theta_{23}-\delta_{230})} & j\bar{E}'_{10}Y_{21}e^{j(\theta_{21}-\delta_{210})} & 0 & -jE'_{30}Y_{23}e^{j(\theta_{23}-\delta_{230})} \\ Y_{31}e^{j(\theta_{31}-\delta_{310})} & Y_{32}e^{j(\theta_{32}-\delta_{320})} & Y_{33}e^{j\theta_{33}} & 0 & -j\bar{E}'_{10}Y_{31}e^{j(\theta_{31}-\delta_{310})} & j\bar{E}'_{20}Y_{32}e^{j(\theta_{32}-\delta_{320})} \end{bmatrix} \begin{bmatrix} \Delta E_1 \\ \Delta E_2 \\ \Delta E_3 \\ \Delta \delta_{12} \\ \Delta \delta_{13} \\ \Delta \delta_{23} \end{bmatrix} \tag{6.15}$$

with generator 1 represented classically, $\bar{E}_1 = 0$ and $\bar{E}'_{10} = \bar{E}_1$; and with node 1 of the classically represented generator, $\bar{E}_1 = E_1 + j0 = E_1$ (a constant).

Substituting this in equation (6.15) and using $\delta_{23} = \delta_{13} - \delta_{12}$, the equation (6.15) becomes

$$
\begin{bmatrix} \Delta \bar{I}_1 \\ \Delta \bar{I}_2 \\ \Delta \bar{I}_3 \end{bmatrix}
=
\begin{bmatrix}
Y_{12}e^{j(\theta_{12}-\delta_{120})} & Y_{13}e^{j(\theta_{13}-\delta_{130})} & -j\bar{E}'_{20}Y_{12}e^{j(\theta_{12}-\delta_{120})} & -j\bar{E}'_{30}Y_{13}e^{j(\theta_{13}-\delta_{130})} \\
Y_{22}e^{j\theta_{22}} & Y_{23}e^{j(\theta_{23}-\delta_{230})} & j\left[E_1Y_{21}e^{j(\theta_{12}+\delta_{120})} + \bar{E}'_{30}Y_{23}e^{j(\theta_{23}-\delta_{230})}\right] & -j\bar{E}'_{30}Y_{23}e^{j(\theta_{23}-\delta_{230})} \\
Y_{32}e^{j(\theta_{23}+\delta_{230})} & Y_{33}e^{j\theta_{33}} & -j\bar{E}'_{20}Y_{23}e^{j(\theta_{23}+\delta_{230})} & j\left[E_1Y_{13}e^{j(\theta_{13}+\delta_{130})} + \bar{E}'_{20}Y_{23}e^{j(\theta_{23}+\delta_{230})}\right]
\end{bmatrix}
\begin{bmatrix} \Delta \bar{E}_2 \\ \Delta \bar{E}_3 \\ \Delta\delta_{12} \\ \Delta\delta_{13} \end{bmatrix}
\tag{6.16}
$$

where $\Delta\bar{I}_i = \Delta I_{qi} + j\Delta I_{di}$

$\Delta E_i = \Delta E_{qi} + j\Delta E_{di}$

$i = 1,2,3$

From equations (6.14) and (6.16), a three-machine system network can be represented in state-space form as

$$\dot{\underline{X}}_m = [A_m]\ \underline{X}_m + [B_m]\ \underline{u}_m \tag{6.17}$$

where,

$$\underline{X}_m = [\Delta\omega_1 \; \Delta E'_{q2} \; \Delta E'_{d2} \; \Delta\omega_2 \; \Delta E'_{q3} \; \Delta E'_{d3} \; \Delta\omega_3 \; \Delta\delta_{12} \; \Delta\delta_{13}]^t$$

$$\underline{u}_m = [\Delta T_{m1} \; \Delta E_{FD2} \; \Delta T_{m2} \; \Delta E_{FD3} \; \Delta T_{m3}]^t$$

The system matrix $[A_m]$ is obtained by substituting the current variables from equation (6.16) in (6.14), and by combining the first two terms of equation (6.14) which then gives the coefficient matrix $[A_m]$ of the state variables $\underline{X}_m$. The matrix $[A_m]$ is given below

$$[A_m] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & a_{15} & a_{16} & 0 & a_{18} & a_{19} \\ 0 & a_{22} & a_{23} & 0 & a_{25} & a_{26} & 0 & a_{28} & a_{29} \\ 0 & a_{32} & a_{33} & 0 & a_{35} & a_{36} & 0 & a_{38} & a_{39} \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & 0 & a_{48} & a_{49} \\ 0 & a_{52} & a_{53} & 0 & a_{55} & a_{56} & 0 & a_{58} & a_{59} \\ 0 & a_{62} & a_{63} & 0 & a_{65} & a_{66} & 0 & a_{68} & a_{69} \\ 0 & a_{72} & a_{73} & 0 & a_{75} & a_{76} & a_{77} & a_{78} & a_{79} \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

(6.18)

where

$$a_{11} = -\frac{D_1}{M_1}$$

$$a_{12} = -\frac{E_1}{M_1}\, y_{11} \;;\; a_{13} = -a_{12}$$

$$a_{15} = -\frac{E_1}{M_1}\, y_{13} \;;\; a_{16} = -a_{15}$$

$$a_{18} = -\frac{E_1}{M_1}\, y_{15}$$

$$a_{19} = -\frac{E_1}{M_1}\, y_{16}$$

$$a_{22} = \frac{1}{T'_{do2}} \left[-1 + (x_{d2} - x'_2)\, B_{22} \right]$$

$$a_{23} = \frac{1}{T'_{do2}} \left[(x_{d2} - x'_2)\, G_{22} \right]$$

$$a_{25} = \frac{1}{T'_{do2}} (x_{d2} - x'_2)\, y_{43}$$

$$a_{26} = \frac{1}{T'_{do2}} (x_{d2} - x'_2)\, y_{33}$$

$$a_{28} = \frac{1}{T'_{do2}} (x_{d2} - x'_2)\, y_{45}$$

$$a_{29} = \frac{1}{T'_{do2}} (x_{d2} - x'_2)\, y_{46}$$

$$a_{32} = -\frac{1}{T'_{qo2}} (x_{q2} - x_2)\, G_{22}$$

$$a_{33} = -\frac{1}{T'_{qo2}} [\, 1 + (x_{q2} - x'_2)\, B_{22}]$$

$$a_{35} = -\frac{1}{T'_{qo2}} (x_{q2} - x'_2)\, Y_{33}$$

$$a_{36} = -\frac{1}{T'_{qo2}} (x_{q2} - x'_2)\, Y_{34}$$

$$a_{38} = -\frac{1}{T'_{qo2}} (x_{q2} - x'_2)\, Y_{35}$$

$$a_{39} = -\frac{1}{T'_{qo2}} (x_{q2} - x'_2)\, Y_{36}$$

$$a_{42} = -\frac{1}{M_2} [\, I_{q20} + E'_{q20} G_{22} + E'_{d20} B_{22}]$$

$$a_{43} = -\frac{1}{M_2} [\, I_{d20} - E'_{q20} B_{22} + E'_{d20} G_{22}]$$

$$a_{44} = -\frac{D_2}{M_2}$$

$$a_{45} = -\frac{1}{M_2} [\, E'_{q20} Y_{33} + E'_{d20} Y_{43}]$$

$$a_{46} = -\frac{1}{M_2} [E'_{q20} Y_{34} + E'_{d20} Y_{44}]$$

$$a_{48} = -\frac{1}{M_2} [E'_{q20} Y_{35} + E'_{d20} Y_{45}]$$

$$a_{49} = - \frac{1}{M_2} \left[E'_{q20} Y_{36} + E'_{d20} Y_{46} \right]$$

$$a_{52} = \frac{1}{T'_{do3}} (x_{d3} - x'_3) \, Y_{61}$$

$$a_{53} = \frac{1}{T'_{do3}} (x_{d3} - x'_3) \, Y_{51}$$

$$a_{55} = \frac{1}{T'_{do3}} \left[-1 + (x_{d3} - x'_3) \, B_{33} \right]$$

$$a_{56} = \frac{1}{T'_{do3}} (x_{d3} - x'_3)$$

$$a_{58} = \frac{1}{T'_{do3}} (x_{d3} - x'_3) \, Y_{65}$$

$$a_{59} = \frac{1}{T'_{do3}} (x_{d3} - x'_3) \, Y_{66}$$

$$a_{62} = - \frac{1}{T'_{qo3}} (x_{q3} - x'_3) \, Y_{51}$$

$$a_{63} = - \frac{1}{T'_{qo3}} (x_{q3} - x'_3) \, Y_{52}$$

$$a_{65} = - \frac{1}{T'_{qo3}} (x_{q3} - x'_3) \, G_{33}$$

$$a_{66} = - \frac{1}{T'_{qo3}} \left[1 - (x_{q3} - x'_3) \, B_{33} \right]$$

$$a_{68} = - \frac{1}{T'_{qo3}} (x_{q3} - x'_3) \, Y_{55}$$

$$a_{69} = -\frac{1}{T'_{qo3}} (x_{q3} - x'_3)\, y_{56}$$

$$a_{72} = -\frac{1}{M_3} [\, E'_{q30}\, y_{51} + E'_{d30}\, y_{61} \,]$$

$$a_{73} = -\frac{1}{M_3} [\, E'_{q30} y_{52} + E'_{d30} y_{51} \,]$$

$$a_{75} = -\frac{1}{M_3} [\, I_{q30} + E'_{q30} G_{33} + E'_{d30} B_{33} \,]$$

$$a_{76} = -\frac{1}{M_3} [\, I_{d30} - E'_{q30} B_{33} + E'_{d30} G_{33} \,]$$

$$a_{77} = -\frac{D_3}{M_3}$$

$$a_{78} = -\frac{1}{M_3} [\, E'_{q30} y_{55} + E'_{d30} y_{65} \,]$$

$$a_{79} = -\frac{1}{M_3} [\, E'_{q30} y_{56} + E'_{d30} y_{66} \,]$$

6.3 CASE STUDY FOR DYNAMIC STABILITY INVESTIGATION

A case study has been conducted for the dynamic stability investigation for the different configurations of a sample network in order to study the performance of six-phase system in comparision to three-phase systems.

6.3.1 A Sample System

A sample network as shown in Fig. 4.2 has been taken for the present investigation at the initial operating point obtained from the load flow study. The loads have been simulated as constant impedances based upon the initial operating conditions. Linearized machine equations have been used. The classical model for generator 1 and the two-axis model for generator nos. 2 and 3 have been used. The data for the 3 synchronous machines are given in Table 6.1. The initial operating conditions are obtained with the help of the data obtained from the load flow study provided in Section 4.4.

6.3.2 Different Configurations of the Sample System under Investigation

The following configurations of the sample system have been considered for the dynamic stability investigation:

1. A completely three-phase system having line nos. 2,3 and 6 as double-circuit three-phase lines
2. A mixed system when line no.2 is converted to a six-phase line
3. A mixed system when both the lines no.2 and 3 are converted to six-phase lines.

TABLE 6.1

Three-Machine System Data

Quantity	unit	Generator 1 (classical)	Generator 2 (two-axis)	Generator 3 (two-axis)
H(MW.S/100MVA)	Second	23.64	6.4	3.0
$M_j = 2H\omega_B$	pu	17824.14	4825.4863	2269.4865
$M_j = 2H/\omega_B$	sec/elect radian	.1254	.03395	.01597
x_d	pu	0.146	0.8958	1.3125
x_d'	pu	0.0608	0.1198	0.1813
x_q	pu	0.0969	0.8645	1.2578
x_q'	pu	0.0969	0.1969	0.25
T_{qo}'	Sec	0.0	0.535	0.6
T_{qo}'	pu	0.0	201.69	226.19
T_{qo}'	sec/elect radian	0.0	.001419	.00159
T_{do}'	sec	8.96	6.0	5.89
T_{do}'	pu	3377.8404	2260.9467	2220.4777
T_{do}'	sec/elect radian	.02376	.01591	.01562

4. A mixed three-phase and six-phase system when all the double circuit three-phase lines i.e., line nos. 2,3 and 6 are converted to single circuit six-phase lines.

The investigations for configurations (2), (3) and (4) have been carried out under two voltage conditions,viz

A when the phase to ground voltage of a six-phase line is $\sqrt{3}$ times that of the double-circuit three-phase lines and

B when the phase to ground voltage of the six-phase line and the double circuit three-phase line are the same.

The dynamic stability investigation has been carried out for the above configurations for case a and case b as described in Section 4.3. The above investigations have been carried out for the same loading of the various configurations of the sample network. Further, a mixed three-phase and six-phase system have been considered with the increased loading i.e., 1.732 times that of the initial loading, when all the double circuit three-phase lines 2,3 and 6 have been converted to single circuit six-phase lines under voltage condition A for case a (i.e. when phase to ground voltage of a six-phase system is $\sqrt{3}$ times that of a three-phase system).

Further, the dynamic stability investigation has been conducted on a completely six-phase system with the same and with the increased loadings having the same phase to

ground voltage as that of the completely three-phase system for case a. In this case obviously, three-phase/six-phase transformers will not be required, rather six-phase transformers will be required in place of three-phase transformers. The inertia constant and the damping coefficient of six-phase synchronous machines have been taken to be equal to those of three-phase synchronous machines. The per phase reactances and the time constants of six-phase machines have also been assumed to be equal to those of three-phase machines.

Analysis

The dynamic stability of the different configurations has been determined with the help of eigenvalue technique. When controlling variables $\underline{u}_m = 0$ in equation (6.17), we have

$$\dot{\underline{x}}_m = [A_m] \underline{x}_m \qquad (6.19)$$

The eigenvalues of the system matrix $[A_m]$ are determined for two damping coefficients of the synchronous machines, namely $D_i = 0$ or 1.0 pu for $i = 1,2,3$.

6.3.3 Results and Discussions

The eigenvalues of system matrix $[A_m]$ for the different configurations of the sample network for two damping coefficients have been obtained and are provided in

Table 6.2 - 6.12. Table 6.2 shows the eigenvalues of system matrix $[A_m]$ corresponding to the original completely three-phase sample network for two damping coefficients of the machines; Table 6.3 to 6.6 shows the eigenvalues for three configurations under voltage conditions A and B for two damping coefficients respectively for case a; and the Tables 6.7 to 6.10, those for case b. In table 6.11, the eigenvalues of system matrix $[A_m]$ corresponding to voltage conditions A of case a are tabulated for configuration III with the increased loading for two damping coefficients of the machines. The results for the completely six-phase system with the same as well as increased loadings to that of the original three-phase system are provided in Table 6.12.

The initial value responses have been obtained using DVERK subroutine and are shown in Figs. 6.2-6.7. Figs. 6.2-6.3 show the responses (δ_{12} and δ_{13} vs. t) for configuration I and configuration III under voltage condition A for case a; figs. 6.4-6.5, those under voltage condition B for case b; figs. 6.6-6.7, those for the completely three-phase system and the completely six-phase system for case a under voltage condition B.

From the comparision of Tables 6.2 - 6.6, it is observed that, as the number of conversion from double-

TABLE 6.2

Eigenvalues of System Matrix $[A_m]$ for a Completely Three-Phase System

Eigenvalues	Damping Coefficients	
	$D_1 = D_2 = D_3 = 0$	$D_1 = D_2 = D_3 = 1.0$
λ_1, λ_2	$-.926 \pm j13.0573$	$1.004 \pm j13.0621$
λ_3, λ_4	$-.1994 \pm j8.664$	$-0.234 \pm j8.665$
λ_5	-6.28	-6.275
λ_6	-3.908	-3.91
λ_7	$-.1058$	$-0.07501 \pm j.0487$
λ_8	0.0	
λ_9	$-.1727$	-0.1715

TABLE 6.3

Eigenvalues of System Matrix $[A_m]$ under voltage Conditions A for case a

(Neglecting damping coefficients of machines)

Eigenvalues	System configurations		
	configuration I	configurations II	configuration III
λ_1, λ_2	$1.0828 \pm j3.857$	$-1.4564 \pm j15.939$	$-1.8756 \pm j18.4579$
λ_3, λ_4	$-0.24067 \pm j9.162$	$-0.4126 \pm j11.031$	$-0.52 \pm j12.27$
λ_5	-5.9948	-5.2408	-4.601
λ_6	-3.76146	-3.3166	-2.8197
λ_7	-0.1188	-0.1521	-0.1714
λ_8	0.0	0.0	0.0
λ_9	-0.1907	-0.2361	-0.2936

System configuration refers to the mixed system having the following meanings :

config. I : when line no. 2 is a six-phase line

config. II : when both the lines nos. 2 and 3 are six-phase lines

config. III : when all the double circuit three-phase lines viz line nos. 2,3 and 6 are six-phase lines.

TABLE 6.4

Eigenvalues of System Matrix $[A_m]$ under Voltage Conditions B for case a

(neglecting damping coefficients of machines)

Eigenvalues	System Configurations		
	configuration I	configuration II	configuration III
λ_1, λ_2	$-.883 \pm j12.8097$	$-.8148 \pm j12.6178$	$-0.715 \pm j12.06$
λ_3, λ_4	$-0.1813 \pm j8.4783$	$-0.1572 \pm j7.974$	$-0.1563 \pm j7.892$
λ_5	-6.3166	-6.405	-6.139
λ_6	-3.8631	-3.8105	-3.815
λ_7	-0.1016	-0.081	-0.07825
λ_8	0.0	0.0	0.0
λ_9	-0.1682	-0.1594	-0.1494

TABLE 6.5

Eigenvalues of System Matrix $[A_m]$ under Voltage Conditions A for case a

(**uniform** damping coefficients 1.0 pu of machines)

Eigenvalues	system configurations		
	configuration I	configuration II	configuration III
λ_1, λ_2	-1.16 ± 13.862	$-1.5337 \pm j15.946$	$-1.9522 \pm j18.455$
λ_3, λ_4	$-0.2754 \pm j9.152$	$-0.4486 \pm j11.0318$	$-0.5569 \pm j12.272$
λ_5	-5.9896	-5.235	-4.5964
λ_6	-3.764	-3.3207	-2.8255
λ_7, λ_8	$-0.0812 \pm j.03796$	$-0.06391, -0.1286$	$-0.1589, -0.0513$
λ_9	-0.1895	-0.23593	-0.2931

TABLE 6.6

Eigenvalues of System Matrix [A_m] under Voltage Conditions B for case a

(assuming uniform damping (1.0 pu) of the machines)

Eigenvalues	System configurations		
	CONFIGURATION I	CONFIGURATION II	CONFIGURATION III
λ_1, λ_2	$-.9596 \pm j12.814$	$-0.8916 \pm j12.522$	$.7922 \pm j12.064$
λ_3, λ_4	$-0.2166 \pm j8.4785$	$-0.1918 \pm j7.9744$	$-0.1902 \pm j7.891$
λ_5	-6.312	-6.402	-6.135
λ_6	-3.866	-3.8136	-3.8182
λ_7, λ_8	$-.0728 \pm j.0525$	$-.0633 \pm j.060172$	$-.0623 \pm j.060547$
λ_9	-0.1669	-0.1576	-0.1471

TABLE 6.7

Eigenvalues of the System Matrix$[A_m]$ under Voltage Condition A for case b

(neglecting damping coefficients of machines)

Eigenvalues	System configurations		
	configuration I	configuration II	configuration III
λ_1, λ_2	$-.7202 \pm j12.136$	$-0.4569 \pm j10.488$	$-0.4198 \pm j10.083$
λ_3, λ_4	$-0.1734 \pm j8.0088$	$-0.2039 \pm j7.1977$	$-0.267 \pm j6.688$
λ_5	-6.73	-7.336	-7.672
λ_6	-4.1226	-4.445	-4.37
λ_7	-0.07554	-0.032	+0.01
λ_8	0.0	0.0	0.0
λ_9	-0.1457	-0.09858	-0.08415

TABLE 6.8

Eigenvalues of System Matrix $[A_m]$ under Voltage Condidition B for case b

(neglecting damping coefficients of machines)

Eigenvalues	system configurations		
	configuration I	configuration II	configuration III
λ_1, λ_2	-0.912 ± 12.9387	$-0.9495 \pm j13.1187$	$-0.852 \pm j12.703$
λ_3, λ_4	$-0.1865 \pm j8.654$	$-0.1724 \pm j8.32$	$-0.1717 \pm j8.287$
λ_5	-6.2578	-6.134	-5.852
λ_6	-3.834	-3.694	-3.685
λ_7	$-.105$	-0.098	-0.0977
λ_8	0.0	0.0	0.0
λ_9	-0.1717	-0.176	-0.16837

TABLE 6.9

Eigenvalues of System matrix [A_m] Under Voltage Condition A for case b

(assuming uniform damping coefficients 1.0 pu of the machines.)

Eigenvalues	System configurations		
	configuration I	configuration II	configuration III
λ_1, λ_2	$-0.7976 \pm j12.14046$	$-0.5329 \pm j10.491$	$-0.495 \pm j10.0867$
λ_3, λ_4	$-0.2069 \pm j8.009$	$-0.2369 \pm j7.1979$	$-0.2996 \pm j6.6887$
λ_5	-6.7265	-7.3346	-7.671
λ_6	-4.1254	-4.4475	-4.378
λ_7, λ_8	$-0.0608 \pm j.06285$	$-0.0397 \pm j.0832$	$-0.02115 \pm j.0922$
λ_9	-0.144	-0.09827	-0.0809

TABLE 6.10

Eigenvalues of System Matrix [A_m] under Voltage Condition B for case b

(assuming uniform damping coefficients (1.0pu) of machines)

Eigenvalues	system configuration		
	configuration I	configuration II	configuration III
λ_1, λ_2	$-0.9887 \pm j12.9433$	$-1.026 \pm j13.1233$	$-0.9293 \pm j12.707$
λ_3, λ_4	$-0.2219 \pm j8.654$	$-0.2074 \pm j8.324$	$-0.2062 \pm j8.287$
λ_5	-6.253	-6.129	-5.8577
λ_6	-3.837	-3.698	-3.688
λ_7, λ_8	$-0.0744 \pm j.05064$	$-0.0714 \pm j.05223$	$-0.0714 \pm j.05178$
λ_9	-0.1705	-0.1745	-0.1664

TABLE 6.11

Eigenvalues of System Matrix $[A_m]$ with the increased System Loading under Voltage Condition A for case a

Eigenvalues	A mixed three-phase and six-phase system with increased loading (Line no. 2,3 and 6 are six-phase lines)	
	$D_1 = D_2 = D_3 = 1$	$D_1 = D_2 = D_3 = 0$
λ_1, λ_2	$-1.23018 \pm j14.0282$	$-1.15239 \pm j14.02258$
λ_3, λ_4	$-0.35373 \pm j9.522$	$-0.31647 \pm j9.5208$
λ_5	-6.17694	-6.1825
λ_6	-3.9176	-3.9112
λ_7	-0.19403	-0.1954
λ_8	-0.1225	-0.1754
λ_9	-0.08883	-0.0

TABLE 6.12

Eigenvalues of System Matrix $[A_m]$ for a completely Six-Phase System under voltage condition B for case a

Eigen-values	with same loading $D_1=D_2=D_3=1.0$	with same loading $D_1=D_2=D_3=0.0$	with increased loading $D_1=D_2=D_3=1.0$	with increased loading $D_1=D_2=D_3=0.0$
λ_1, λ_2	$-1.9954 \pm j13.82$	$-1.9173 \pm j13.81$	$-1.7175 \pm j12.12$	$1.638 \pm j12.115$
λ_3, λ_4	$-0.6036 \pm j8.61$	$-0.5669 \pm j8.61$	$-0.5496 \pm j7.897$	$-0.5132 \pm j7.896$
λ_5	-3.3828	-3.3889	-4.05702	-4.064
λ_6	-2.391	-2.391	-2.7284	-2.7263
λ_7	-0.05026	-0.0	-0.05473	-0.0
λ_8	-0.3765	-0.37754	-0.30698	-0.21794
λ_9	-0.21657	-0.22356	-0.204318	-0.3096

circuit three-phase lines to single-circuit six-phase lines under voltage condition A (i.e., both six-phase and three-phase system having same line to line voltage) are increased, the eigenvalues moves towards left. But when the same conversion is carried out under voltage condition B (i.e. both six-phase and three-phase system having same phase to ground voltage), the eigenvalues shift towards right. It thus reveals that the conversion to the six-phase system increases the dynamic stability as the number of conversion increases under voltage configuration A while it decreases under voltage configuration B with the loading as that of the original three-phase system. Further, when Tables 6.7 - 6.10 are observed, where the results for case b with the same loading as the previous one are tabulated, it shows that the eigenvalues are shifting right thereby the system being less dynamically stable irrespective of the voltage conditions A and B with the fixed loading.

When the results of Table 6.11 are compared to those of Tables 6.2, and to those of Tables 6.3 and 6.5 under configuration III, the eigenvalues of the system matrix $[A_m]$ corresponding to the increased loading system under configuration III for voltage condition A have shifted towards right as compared to fixed loading and, hence, the power system is less dynamically stable with increased loading than with

the same loading. If, however, the eigenvalues corresponding to configuration III of the system with increased loading is compared to those of the completely three-phase system with the original loading, the system with the increased loading is more dynamically stable than the original three-phase system.

When the results of Table 6.12 are compared with those of Tables 6.2, 6.4 and 6.6, it is observed that a completely six-phase system with the same loading and also with the increased loading is more dynamically stable than a completely three-phase system. It is also observed that, with respect to system configuration under voltage condition B, a completely six-phase system with the same loading or with increased loading is dynamically more stable than a completely three-phase system. Further, comparisions of Table 6.12 with tables 6.3 and 6.5 show that the completely six-phase system under voltage condition B is dynamically more stable than a mixed system under configuration III for voltage condition A for the same loading, while the former with increased loading is dynamically less stable than the latter.

From the initial value responses ($\Delta\delta_{12}$, $\Delta\delta_{13} \sim t$) shown in Figs. 6.2 - 6.3, it is observed that the dynamic

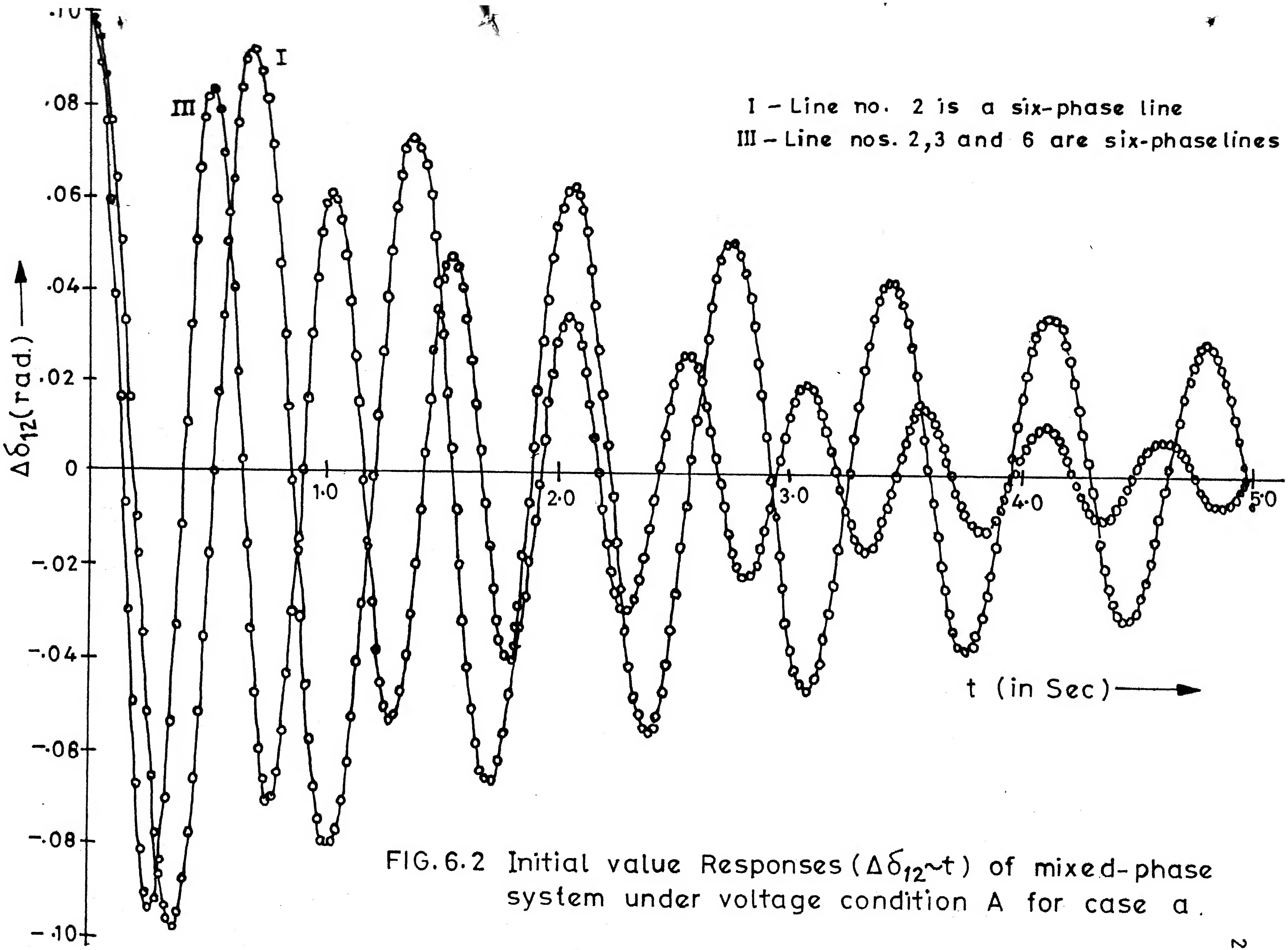


FIG. 6.2 Initial value Responses ($\Delta\delta_{12} \sim t$) of mixed-phase system under voltage condition A for case a.

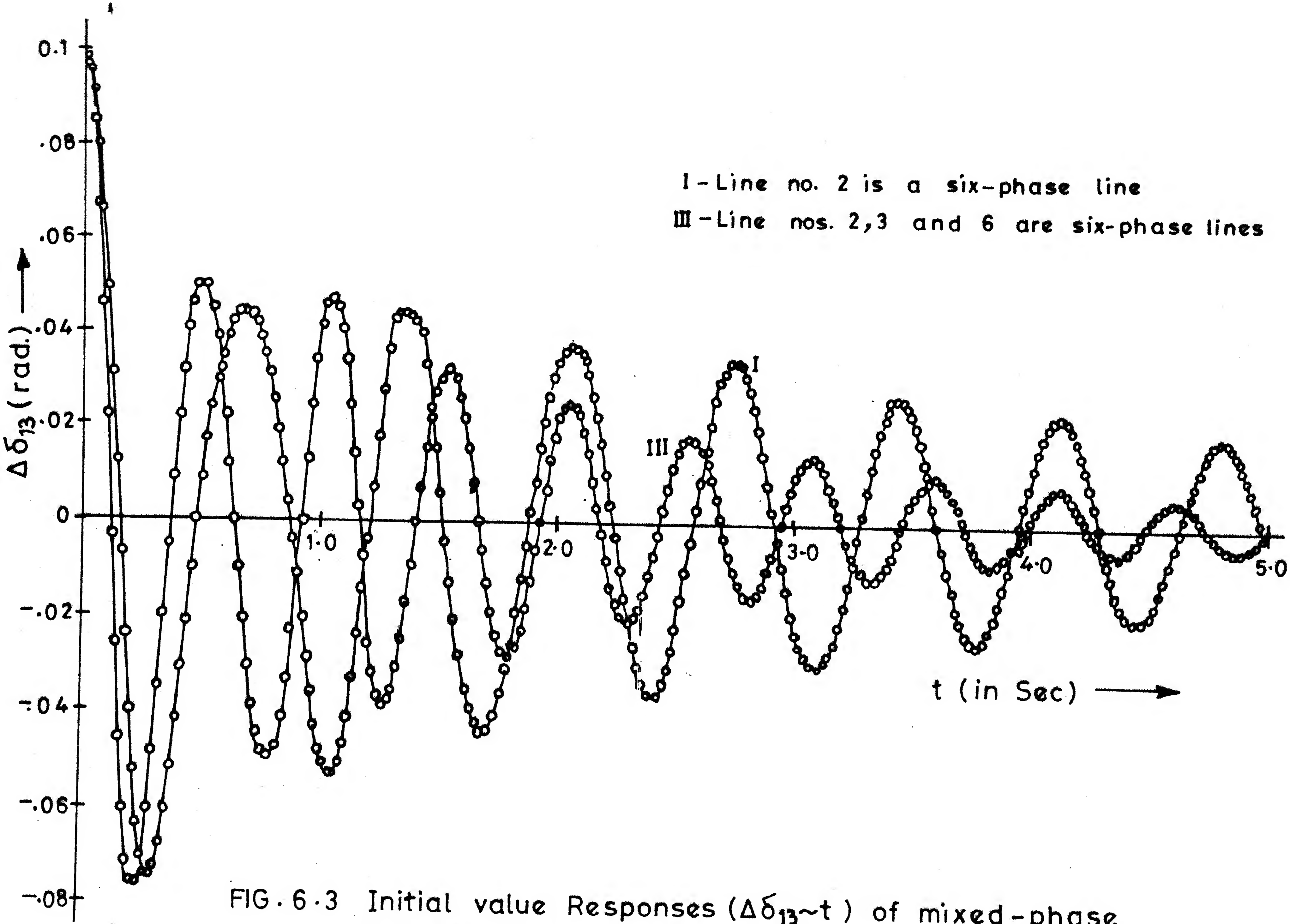


FIG. 6.3 Initial value Responses ($\Delta\delta_{13} \sim t$) of mixed-phase system under voltage condition A for case a.

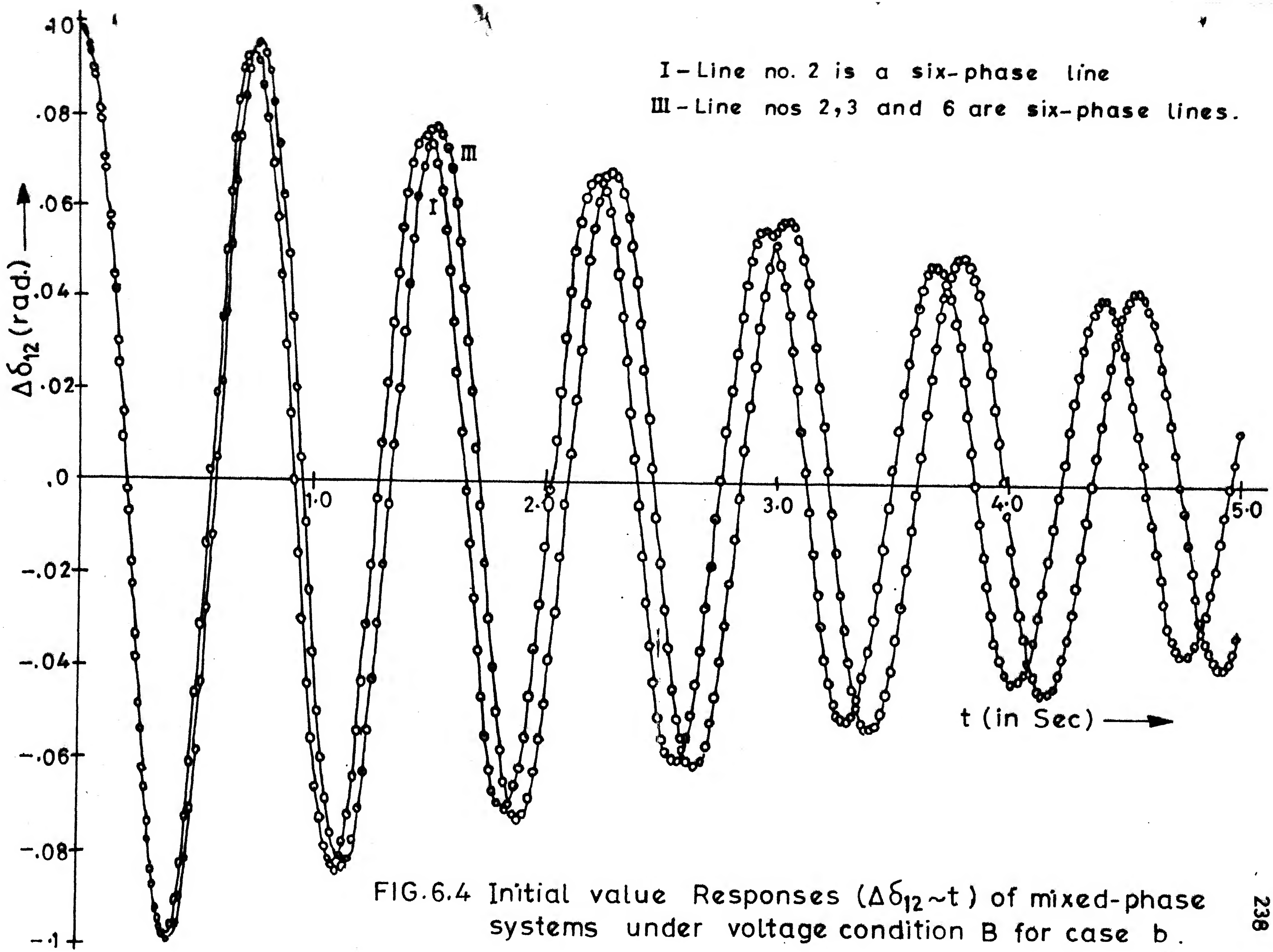


FIG.6.4 Initial value Responses ($\Delta\delta_{12} \sim t$) of mixed-phase systems under voltage condition B for case b.

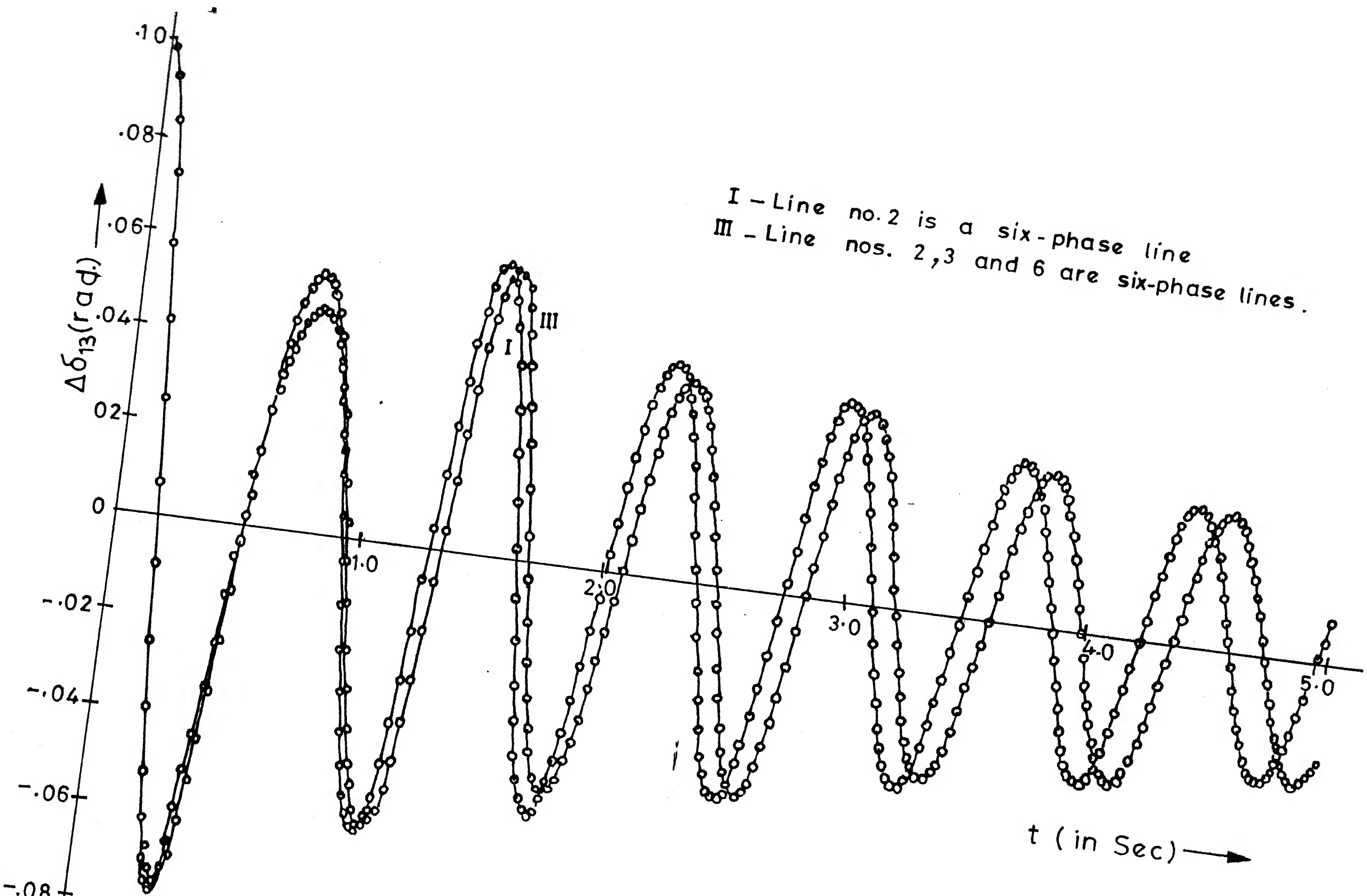


FIG. 6.5 Initial value Responses ($\Delta\delta_{13} \sim t$) of mixed-phase systems under voltage condition B for case b.

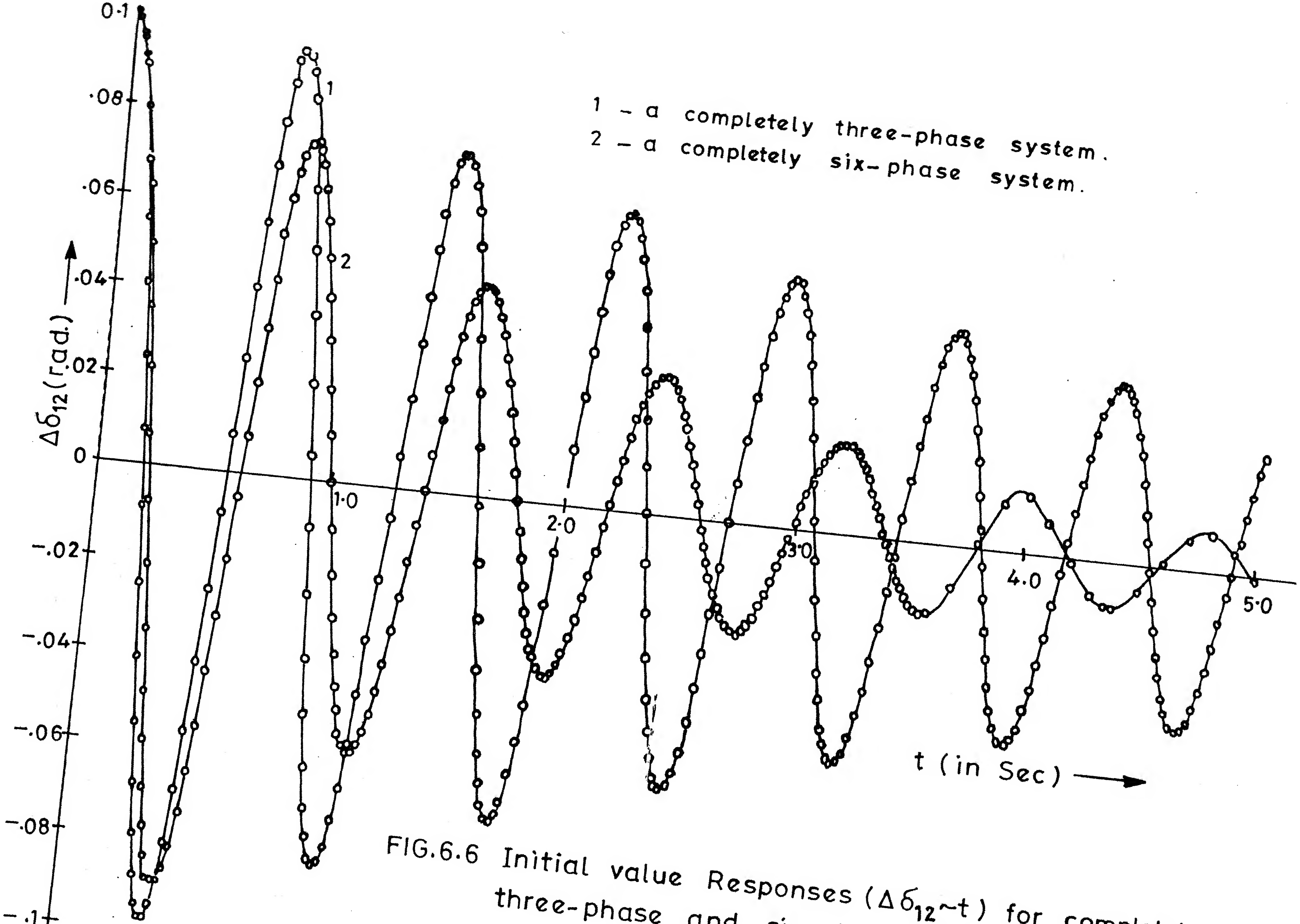


FIG.6.6 Initial value Responses ($\Delta\delta_{12} \sim t$) for completely three-phase and six-phase systems.

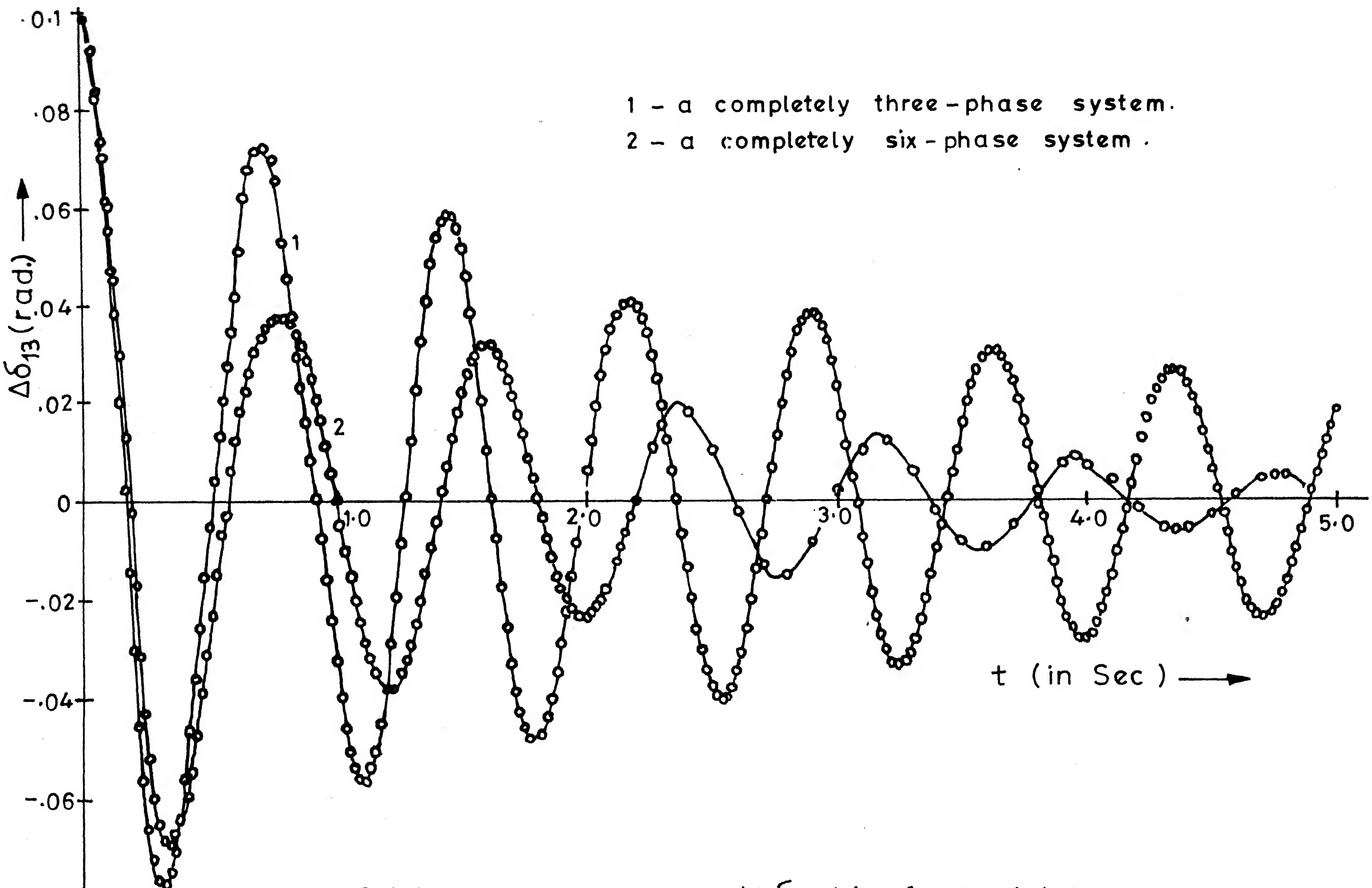


FIG.6.7 Initial value Responses ($\Delta\delta_{13} \sim t$) of completely three-phase and six-phase systems.

stability of the system has considerably improved with the increasing conversions of double circuit three-phase lines to single circuit six-phase lines under voltage condition A for case a. But the dynamic stability of the mixed system under voltage condition B for case b as observed from the initial responses (figs. 6.4 - 6.5) has not been affected, it has rather slightly reduced with increasing conversions. Further, it is observed from Figs. 6.6 and 6.7 that a completely six-phase system is better than a completely three-phase system from the stability point of view.

6.4 CONCLUSION

A dynamic stability investigation has been carried out for different system configurations in order to observe the relative performances of a completely three-phase system, a mixed three-phase and six-phase system, and a completely six-phase system. It is observed that a mixed three-phase and six-phase system improves the stability with the increasing number of conversions from double circuit three-phase lines to single-circuit six-phase lines with the same line to line (i.e. adjacent phase) voltage, but the system becomes less stable with the increasing conversions taking

place at the samephase to ground voltage. Further, it is observed that a completely six-phase system with the same or increased system loading is dynamically more stable than a completely three-phase system.

CHAPTER 7

CONCLUSIONS

The mathematical modelling for the various multi-phase elements of power system network has been developed and based upon these models, various studies such as load flow, transient and dynamic stability studies on a sample network for its different configurations have been conducted with a view to investigate the performance of a mixed phase as well as a completely multi-phase i.e., completely six-phase system in comparision to that of a conventional three-phase system.

The inductances of a six-phase machine in the phasor form have been obtained from the fundamentals. A detailed dynamic model of a six-phase machine has been developed with the help of an orthogonal transformation. Further, a linearized model of the machine has been developed in terms of current variables. The dynamic stability of a six-phase machine has been investigated on a sample system, employing eigenvalue technique, the results of the six-phase synchronous machine have been compared to those of the three-phase machine.

Further, the mathematical model of a multi-phase (n-phase) synchronous machine in the phasor form has been developed and the corresponding model of a six-phase as well

as twelve phase machine has been derived. For the purpose of n-phase transmission, three-phase/n-phase transformers are necessary to be compatible with the system; three-phase/n-phase transformer models for different winding connections have been developed with off-nominal turn-ratios on primary as well as on secondary side.

The three-phase equivalent of a multi-phase (n-phase) line connected through two three-phase/n-phase transformers has been derived in terms of three-phase impedance matrix and the three-phase ABCD parameters for π circuit representation, and further based upon the three-phase equivalent representation, a single phase equivalent has been obtained. The n-phase equivalent of a three-phase line integrated with two three-phase/n-phase transformers has also been derived.

Further, two types of problems have been formulated and investigated for the load flow analysis based upon one line diagram. The equivalent single-phase representation has been derived for the six-phase and twelve-phase transmission lines with three-phase/six-phase or three-phase/twelve-phase transformers at both the ends of multi-phase lines, which also helps in obtaining the parameters of single-phase equivalent in pu on the same base as that of three-phase line. The load flow investigation has been carried out on a sample

network for its different configurations obtained by converting double-circuit three-phase lines to multi-phase (six-phase or twelve-phase) lines with the phase to ground voltage equal to, or $\sqrt{3}$ times as that of the double-circuit three-phase lines.

The direct and the quadrature axis transient and subtransient inductances of a six-phase machine have been obtained from the fundamentals, and the different time constants for a six-phase machine have also been calculated. The transient stability of a sample network has been investigated to study the impact of converting double-circuit three-phase lines to single-circuit six-phase lines for the two cases as outlined in the load flow study. A completely six-phase system with the same voltage has also been investigated for transient stability studies.

The dynamic stability of a multi-machine system has been carried out on a sample network by conducting investigation for the different configurations of the sample network. The relative performances of mixed-phase systems, completely six-phase system and a conventional three-phase system have also been studied.

Based upon the case studies for mixed-phase and six-phase (multi-phase) systems, the performances regarding high-phase order have been observed which are discussed briefly in the following paragraphs.

The six-phase machine is dynamically more stable than the three-phase machine when the same power is delivered by both the machines. When the six-phase machine delivers double the power, it shows a little improvement in stability as compared to that of a three-phase machine.

The magnitude of bus voltages of mixed-phase systems having the same slack bus voltage increases or decreases with the increasing number of conversions of double-circuit three-phase lines with equal line to line (i.e., adjacent phase) voltage or equal phase to ground voltage. When the same reactive power at the slack bus is considered, the bus voltages of the mixed-phase system for the same system loading increase or decrease with the same phase to ground voltage or with the same line to line (i.e. adjacent phase) voltage as that of the double-circuit three-phase lines, with the increasing conversions of double-circuit three-phase lines to single-circuit six-phase lines.

The transient stability of mixed-phase system is increased with the same as well as with increased loading to that of the completely three-phase system, when the double-circuit three-phase lines are converted to six-phase lines with the same line to line (i.e., adjacent phase) voltages, but if the conversion takes place with the same line to

neutral voltage with the same loading, the stability reduces. Also, the stability increases when a three-phase system is converted to a completely six-phase system with equal line to neutral voltage for the same loading as well as for increased loading. It has further been observed that the impact of the conversion on the stability depends upon the location of the converted line from the fault.

The dynamic stability of multi-machine power system shows that a mixed-phase system is dynamically more stable than the three-phase system with increasing conversions, when the line to line (adjacent phase) voltages of the six-phase and the double-circuit three-phase lines are the same; the stability decreases with equal line to neutral voltage. When mixed-phase system is considered with increased loading, the system is more stable than a completely three-phase system. For a completely six-phase system with equal and increased loading, it shows an improvement in stability as compared to that of a three-phase system.

Based upon the investigation and results, we now give the future scope of the work in this area.

Future Scope of Work

A three-phase/multi-phase transformer is one of the important elements of the mixed-phase systems; so its mathematical modelling, an important aspect in the analysis. A simple model of the transformer representing it by a series leakage impedance along with tappings has been employed in the analysis. It is however desirable to develop its model including details, such as non-linearities caused by saturation, phase shifting effects, etc.

The multi-phase lines have been assumed throughout the study as transposed lines, though actually these are untransposed. It is difficult to achieve the complete transposition of the multi-phase lines, it would however be realistic to analyse the multi-phase with mutual coupling by taking the lines as untransposed. Further, it would be more realistic, if the performance of mixed-phase and the completely multi-systems (i.e. completely six-phase or twelve-phase systems) would be examined having based on the practical data obtained from the designs of multi-phase lines for various conductor configurations and tower geometries.

A classical model of the synchronous machine for the purpose of transient stability study has been employed

where the synchronous machine is represented as a constant voltage source behind the direct-axis transient reactance, and the loads by constant impedances. It is desirable to employ the machine models incorporating such details as the effect of field flux decay, saturation, effects of excitation control and governing systems etc. Also it would be more realistic if the loads would be represented more accurately, such as dependence of loads on frequency, etc.

Although experimental multi-phase lines have been constructed and the field testing and the simulation studies have been reported, yet there is a need to extend experimentation in order to gain the operational experience, and to be acquainted further with the newer problems as well.

The dynamic stability investigation of a six-phase synchronous machine has been conducted by taking its parameters based upon those of three-phase machine. It is, however, desirable to carry out the investigation based upon the actual parameters from its practical design and tests data.

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APPENDIX A

LINE PARAMETERS FOR A THREE-BUS SINGLE-MACHINE SAMPLE NETWORK (FIG. 4.1)

TABLE A.1

Line Parameters and Various Capabilities of Different Types of 138 kV Lines of Sample System Shown in Fig.4.1

Type of line	Series Imp. Ω /phase/ mile	Shunt admi. μs/ph/mile	SIL (MW)	Thermal capacity (MW)
Single-circuit line	0.1185+ j0.789	5.387	49.76	185.48
Double-circuit line	0.05925+ j0.353	12.077	111.39	370.96
Six-Phase Line	0.1185+ j0.744	5.7248	316.96	642.53

Lenth of each line = 50 miles.

TABLE A.2

p.u. Line Parameter of Different Types of 138 kV lines on 100MVA Base of a Sample System in Fig. 4.1

Type of line	Series Impedance Z_P	half shunt admittance ($1/2\ Y_{sh}$)
Single circuit three-phase line	.0311+j.2071	0.0+j.0257
Double-circuit three-phase line	.0155+j.09268	0.0+j.0575
Single-circuit six-phase line	.0311+j.1953	0.0+j.0273

APPENDIX B

PARAMETERS OF A NINE-BUS THREE-MACHINE SAMPLE NETWORK (Fig. 4.2)

TABLE B.1

p.u. Line Parameters of the Sample Network

Line No.	Buses From	To	Types(*) of line	Series Impedance Z_{pq}	Half line changing admittance 1/2 Y_{sh}
1	4	5	1	.01+j.085	0.0+j0.088
2	4	6	2	.017+j0.092	0.0+j0.079
3	5	7	2	.032+j0.161	0.0+j0.153
4	6	9	1	.039+j0.17	0.0+j0.179
5	7	8	1	.0085+j.072	0.0+j0.0745
6	8	9	2	.0119+j.1008	0.0+j0.1045

* Types of line in column 3 have the following meanings

1 - Single-circuit three-phase line

2 - Double-circuit three-phase line

TABLE B.2

Pu Transformer Parameters of a Sample System Shown in Fig. 4.2

No.	connecting Bus From	To	Reactance	off-nominal ratio
1	1	4	.0576	1.0
2	2	7	.0625	1.0
3	3	9	.0586	1.0

APPENDIX C

LINEARIZED MODEL FOR MULTIMACHINE SYSTEM WITH CONSTANT IMPEDANCE LOADS

Let $\bar{V}_i$ and $\bar{I}_i$ be the voltages and currents of synchronous machines where $i = 1,2,\ldots,m$; m is the number of synchronous machines in the network. The complex vectors $\bar{V}$ and $\bar{I}$ can be expressed as,

$$\bar{V} = \begin{bmatrix} V_{qi} + j\, V_{d1} \\ V_{q2} + j\, V_{d2} \\ \vdots \qquad \vdots \\ V_{qm} + j\, V_{dm} \end{bmatrix} = \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \vdots \\ \bar{V}_m \end{bmatrix} \qquad (C.1)$$

$$\bar{I} = \begin{bmatrix} I_{q1} + j\, I_{d1} \\ I_{q2} + j\, I_{d2} \\ \vdots \qquad \vdots \\ I_{qm} + j\, I_{dm} \end{bmatrix} = \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_m \end{bmatrix} \qquad (C.2)$$

where the axis q_i is taken as the phasor reference in each case. Let us consider a power system network with m machine buses and r load buses. Since the loads are represented as constant impedances, the network has only m nodes connected with the active sources i.e. synchronous machines. The reduced network is shown in Fig. C.1 For this

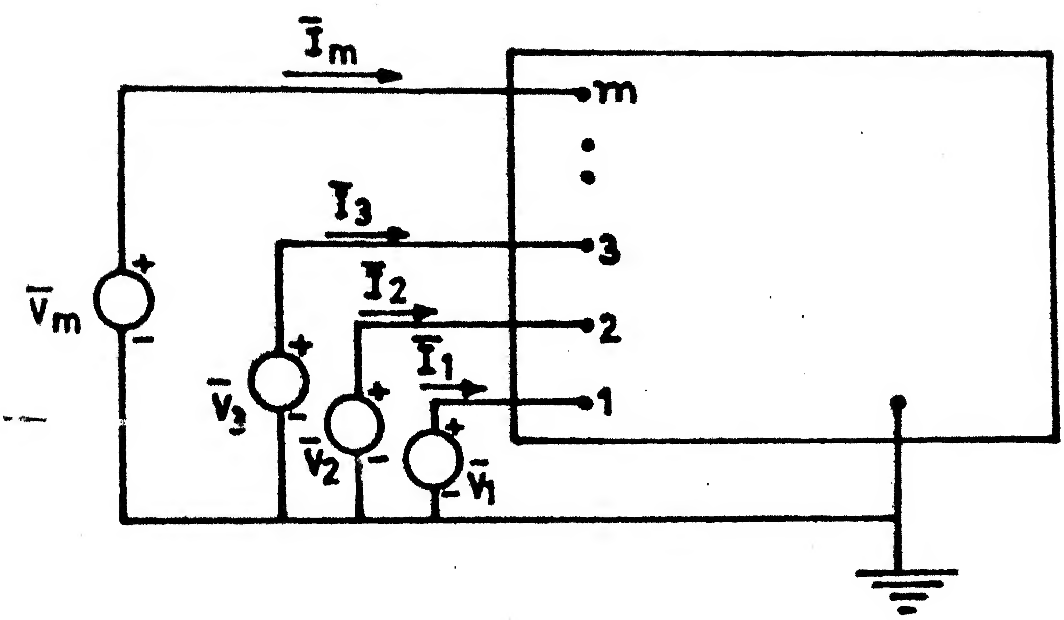

FIG. C.1 A Reduced m-port Network .

network, the node voltages and currents are expressed in phasor notations and these are $\bar{V}_1$, $\bar{V}_2, \ldots, \bar{V}_m$ and $\bar{I}_I, \bar{I}_2, \ldots, \bar{I}_m$ respectively. At steady-state, these currents and voltages can be represented by phasor w.r.t. a common (network) frame of reference and threir relations are given by,

$$\hat{I} = \bar{Y}\hat{V} \tag{C.3}$$

where,

$$\hat{I} = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_m \end{bmatrix}; \quad \hat{V} = \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_m \end{bmatrix}$$

and $\bar{Y}$ is the short-circuit admittance matrix of the reduced network. The network in the transient state can be described by equation (C.3). Similar to those describing its steady-state behaviour under the following assumptions :

a) the system angular speed does not depart appreciably from the rated (i.e. synchronous) speed i.e., $\omega = \omega_R$

b) the transformer voltages are negligible in comparision to the speed voltages.

Conversion of Machine Co-ordinates to System Reference

Let us consider a voltage $\bar{V}_{abci}$ at any node i. It is desired to obtain V_{dqi} from V_{abci} by applying Park's transformation. V_{dqi} is expressed in phasor notation as $\bar{V}_i$ from (C.1) using the rotor magnetic axis of machine i as reference axis. It can also be expressed to the system reference as $\hat{V}_i$ using the transformation

$$\hat{V}_i = \bar{V}_i \, e^{j\delta_i} \tag{C.4}$$

where δ_i is the angle between this rotor magnetic axis and a synchronously rotating reference frame. The rotor angle θ_i of this machine is given by

$$\theta_i = \omega_R t + \frac{\pi}{2} + \delta_i \tag{C.5}$$

Equation (C.4) can be generalized to include all the nodes, and the generalized transformation is

$$T = \begin{bmatrix} e^{j\delta_1} & 0 & \cdot\cdot & 0 \\ 0 & e^{j\delta_2} & \cdot\cdot & 0 \\ \vdots & \vdots & \vdots\vdots & \vdots \\ 0 & 0 & \cdot\cdot & e^{j\delta_m} \end{bmatrix} \tag{C.6}$$

Let $$\hat{V} = \begin{bmatrix} V_{Q1}+jV_{D1} \\ V_{Q2}+jV_{D2} \\ V_{Qm}+jV_{Dm} \end{bmatrix} \tag{C.7}$$

Then from equations (C.1) and (C.4)-(C.7), we have,

$$\hat{V} = T\,\bar{V} \tag{C.8}$$

Thus T is a transformation which transforms the d and q quantities of all machines to the synchronously rotating system frame of reference. T is an orthogonal transformation i.e., $T^{-1} = T^{*}$.

From equation no. (C.8),

$$\bar{V} = T^{*}\,\hat{V} \text{ and similarly } \hat{I} = T\,\bar{I}\;;\;\bar{I} = T^{*}\,\hat{I} \tag{C.9}$$

<u>Relation between Machine Currents and Voltages</u>

From equation (C.3), using (C.8) and (C.9),

$$T\,\bar{I} = \bar{Y}\,T\,\bar{V}$$

i.e. $$\bar{I} = \bar{M}\,\bar{V} \tag{C.10}$$

where $$\bar{M} = (T^{-1}\,\bar{Y}\,T) \tag{C.11}$$

where the matrix $\overline{Y}$ of the network is of the form

$$\overline{Y} = \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j\theta_{12}} & \cdots & Y_{1m}e^{j\theta_{1m}} \\ Y_{21}e^{j\theta_{21}} & Y_{22}e^{j\theta_{22}} & \cdots & Y_{2m}e^{j\theta_{2m}} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{m1}e^{j\theta_{m1}} & Y_{m2}e^{j\theta_{m2}} & \cdots & Y_{mm}e^{j\theta_{mm}} \end{bmatrix} \qquad (C.12)$$

and

$$\overline{M} = \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j(\theta_{12}-\delta_{12})} & \cdots & Y_{1m}e^{j(\theta_{1m}-\delta_{1m})} \\ Y_{21}e^{j(\theta_{21}-\delta_{21})} & Y_{22}e^{j\theta_{22}} & \cdots & Y_{2m}e^{j(\theta_{2m}-\delta_{2m})} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{m1}e^{j(\theta_{m1}-\delta_{m1})} & Y_{m2}e^{j(\theta_{m2}-\delta_{m2})} & \cdots & Y_{mm}e^{j\theta_{mm}} \end{bmatrix} \qquad (C.13)$$

Thus equation (C.10) gives the desired relation between the terminal voltages and currents of the machines.

Linearized Model for the Network

Linearizing equation (C.10), we have,

$$\Delta \overline{I} = \overline{M}_o \Delta \overline{V} + \Delta M \ \overline{V}_o \qquad (C.14)$$

where $\overline{M}_o$ is evaluated at the initial angles δ_{io}, $i = 1,2, \ldots m$, and $\overline{V}_o$ is the initial value of the vector $\overline{V}$.

Let $\delta_i = \delta_{io} + \Delta\delta_i$, then the matrix $\overline{M}$ in equation (C.13) becomes

$$\overline{M} = \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j(\theta_{12}-\delta_{12o}-\Delta\delta_{12})} & \cdots & Y_{1m}e^{j(\theta_{1m}-\delta_{1mo}-\Delta\delta_{1m})} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{m1}e^{j(\theta_{m1}-\delta_{m1o}-\Delta\delta_{m1})} & Y_{m2}e^{j(\theta_{m2}-\delta_{m2o}-\Delta\delta_{m2})} & \cdots & Y_{mm}e^{j\theta_{mm}} \end{bmatrix} \tag{C.15}$$

The general term $\overline{m}_{ij}$ of the matrix $\overline{M}$ is $Y_{ij}e^{j(\theta_{ij}-\delta_{ijo}-\Delta\delta_{ij})}$

Thus $\overline{m}_{ij}$ of the matrix $\overline{M}$ is $Y_{ij}e^{j(\theta_{ij}-\delta_{ijo})}e^{-j\Delta\delta_{ij}}$ (C.16)

Substituting $\cos\Delta\delta_{ij} = 1$ and $\sin\Delta\delta_{ij} \cong \Delta\delta_{ij}$, (C.16) becomes,

$$\overline{m}_{ij} \cong Y_{ij}e^{j(\theta_{ij}-\delta_{ijo})}(1-j\Delta\delta_{ij}) \tag{C.17}$$

Therefore,

$$\Delta\overline{m}_{ij} \cong -j\,Y_{ij}e^{j(\theta_{ij}-\delta_{ijo})}\Delta\delta_{ij} \tag{C.18}$$

$$\text{Hence, } \Delta M\bar{V}_o = -j \begin{bmatrix} \sum_{k=1}^{m} Y_{1k} e^{j(\theta_{1k}-\delta_{1ko})} & \bar{V}_{ko} & \Delta\delta_{1k} \\ \sum_{k=1}^{m} Y_{2k} e^{j(\theta_{2k}-\delta_{2ko})} & \bar{V}_{ko} & \Delta\delta_{2k} \\ \sum_{k=1}^{m} Y_{mk} e^{j(\theta_{mk}-\delta_{mko})} & \bar{V}_{ko} & \Delta\delta_{mk} \end{bmatrix} \qquad (C.19)$$

Thus, the linearized equation (C.14) becomes,

$$\begin{bmatrix} \Delta\bar{I}_1 \\ \Delta\bar{I}_2 \\ \cdot\cdot \\ \Delta\bar{I}_m \end{bmatrix} = \begin{bmatrix} Y_{11} e^{j\theta_{11}} & \cdots & Y_{1m} e^{j(\theta_{1m}-\delta_{1mo})} \\ Y_{21} e^{j(\theta_{21}-\delta_{21o})} & \cdots & Y_{2m} e^{j(\theta_{2m}-\delta_{2mo})} \\ \cdots\cdots & \cdots & \cdots\cdots \\ Y_{m1} e^{j(\theta_{n1}-\delta_{n1o})} & \cdots & Y_{mm} e^{j\theta_{mm}} \end{bmatrix} \begin{bmatrix} \Delta\bar{V}_1 \\ \Delta\bar{V}_2 \\ \\ \Delta\bar{V}_m \end{bmatrix}$$

$$-j \begin{bmatrix} \sum_{k=1}^{m} \bar{V}_{ko} Y_{1k} e^{j(\theta_{1k}-\delta_{1ko})} \\ \sum_{k=1}^{m} \bar{V}_{ko} Y_{2k} e^{j(\theta_{2k}-\delta_{2ko})} \\ \cdots\cdots \quad \cdots\cdots \\ \sum_{i=1}^{m} \bar{V}_{ko} Y_{mk} e^{j(\theta_{mk}-\delta_{mko})} \end{bmatrix} \begin{bmatrix} \Delta\delta_{1k} \\ \Delta\delta_{2k} \\ \cdots\cdot \\ \Delta\delta_{mk} \end{bmatrix} \qquad (C.20)$$

The set of equations (C.20) gives the complete description of the system.

Now the above equation (C.20) has been considered for the sample system with three synchronous machines.

Separating real and imaginary parts of equation (C.20) we have,

$$\begin{bmatrix} \Delta I_{q1} \\ \Delta I_{d1} \\ \Delta I_{q2} \\ \Delta I_{d2} \\ \Delta I_{q3} \\ \Delta I_{d3} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \\ G_{22} & -B_{22} & y_{33} & y_{34} & y_{35} & y_{36} \\ B_{22} & G_{22} & y_{43} & y_{44} & y_{45} & y_{46} \\ y_{51} & y_{52} & G_{33} & -B_{33} & y_{55} & y_{56} \\ y_{61} & y_{62} & B_{33} & G_{33} & y_{65} & y_{66} \end{bmatrix} \begin{bmatrix} \Delta E'_{q2} \\ \Delta E'_{d2} \\ \Delta E'_{q3} \\ \Delta E'_{d3} \\ \Delta \delta_{12} \\ \Delta \delta_{13} \end{bmatrix} \tag{C.21}$$

where,

$$y_{11} = y_{22} = Y_{12} \cos(\theta_{12}-\delta_{120}); \; y_{21} = -y_{12} = Y_{12}\sin(\theta_{12}-\delta_{120})$$

$$y_{13} = y_{24} = Y_{13} \cos(\theta_{13}-\delta_{130}); \; y_{23} = -y_{14} = Y_{13} \sin(\theta_{13}-\delta_{130})$$

$$y_{15} = Y_{12}[E'_{d20} \cos(\theta_{12}-\delta_{120})+E'_{q20} \sin(\theta_{12}-\delta_{120})]$$

$$y_{16} = Y_{13}[E'_{d30} \cos(\theta_{13}-\delta_{130}) + E'_{q30} \sin(\theta_{13}-\delta_{130})]$$

$$y_{25} = Y_{12}[E'_{d20} \sin(\theta_{12}-\delta_{120}) - E'_{q20} \cos(\theta_{12}-\delta_{120})]$$

$$Y_{26} = Y_{13}[E'_{d30} \sin(\theta_{13}-\delta_{130}) - E'_{q30} \cos(\theta_{13}-\delta_{130})]$$

$$Y_{33} = Y_{44} = Y_{23} \cos(\theta_{23}-\delta_{230}); \; Y_{43} = -Y_{34} = Y_{23} \sin(\theta_{23}-\delta_{230})$$

$$Y_{35} = -E_1 Y_{12} \sin(\theta_{12}+\delta_{120}) - E'_{d30}Y_{23} \cos(\theta_{23}-\delta_{230})$$

$$- E_{q30}Y_{23} \sin(\theta_{23} - \delta_{230})$$

$$Y_{36} = Y_{23}[E'_{d30} \cos(\theta_{23}-\delta_{230}) + E'_{q30} \sin(\theta_{23} - \delta_{230})]$$

$$Y_{45} = E_1Y_{12} \cos(\theta_{12}+\delta_{120}) - E'_{d30}Y_{23} \sin(\theta_{23}-\delta_{230})$$

$$+ E'_{q30}Y_{23} \cos(\theta_{23}-\delta_{230})$$

$$Y_{46} = Y_{23} [E'_{d30} \sin(\theta_{23}-\delta_{230}) - E'_{q30} \cos(\theta_{23}-\delta_{230})]$$

$$Y_{51} = Y_{62} = Y_{23} \cos(\theta_{23}+\delta_{230}); \; Y_{61} = - Y_{52} = Y_{23} \sin(\theta_{23}+\delta_{230})$$

$$Y_{55} = Y_{23}[E'_{d20} \cos(\theta_{23}+\delta_{230})+E'_{q20} \sin(\theta_{23} + \delta_{230})]$$

$$Y_{56} = -E_1Y_{13} \sin(\theta_{13} + \delta_{130}) - Y_{23} E'_{d20} \cos(\theta_{23}+ \delta_{230})$$

$$-Y_{23} E'_{q20} \sin(\theta_{23} + \delta_{230})$$

$$Y_{65} = Y_{23}[E'_{d20} \sin(\theta_{23} + \delta_{230}) - E'_{q20} \cos(\theta_{23} + \delta_{230})]$$

$$Y_{66} = E_1Y_{13} \cos(\theta_{13} + \delta_{130}) - E'_{d20}Y_{23} \sin(\theta_{23} + \delta_{230})$$

$$+ E'_{q20}Y_{23} \cos(\theta_{23} + \delta_{230})$$

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2. 'Operating Point Stability Analysis of a Six-Phase Synchronous Machine' (With Dr. L.P. Singh) Accepted for the presentation at International Federation of Automatic Control (IFAC) to be held from 15-17 December, 1986.

3. 'Six-Phase (Multi-Phase) Systems Dynamic Modelling and Transient Stability Analysis' (With Dr. L.P. Singh) Communicated to 'Electric Power System Research'.

* 4. 'Design of a Dynamic Excitation Controller for Wide-Speed Range Operation of a Synchronous Motor Drive' (With Dr. S.S. Prabhu) presented at the IV National Power System Conference held in 1986 at VARANASI.

* It is not pertaining to the Ph.D. work.